NANYANG JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME

CLASS

TUTOR'S NAME: Solution

PHYSICS

9646/03

Paper 3 Longer Structured questions

23 September 2014

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Section A
Answer **all** questions.

Section B
Answer any **two** questions.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use	
Section A	
1	
2	
3	
4	
5	
Section B	
6	
7	
8	
deductions	
Total	**80**

Data

speed of light in free space,	c	=	3.00×10^{8} m s^{-1}
permeability of free space,	μ_o	=	$4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space,	ε_o	=	8.85×10^{-12} Fm^{-1}
			$(1 / (36\pi)) \times 10^{-9}$ Fm^{-1}
elementary charge,	e	=	1.60×10^{-19} C
the Planck constant,	h	=	6.63×10^{-34} J s
unified atomic mass constant,	u	=	1.66×10^{-27} kg
rest mass of electron,	m_e	=	9.11×10^{-31} kg
rest mass of proton,	m_p	=	1.67×10^{-27} kg
molar gas constant,	R	=	8.31 J K^{-1} mol^{-1}
the Avogadro constant,	N_A	=	6.02×10^{23} mol^{-1}
the Boltzmann constant,	k	=	1.38×10^{-23} J K^{-1}
gravitational constant,	G	=	6.67×10^{-11} N m^{2} kg^{-2}
acceleration of free fall,	g	=	9.81 m s^{-2}

Formulae

uniformly accelerated motion,	s	=	$ut + \frac{1}{2}at^2$
	v^2	=	$u^2 + 2as$
work done on/by a gas,	W	=	$p\Delta V$
hydrostatic pressure,	p	=	ρgh
gravitational potential,	ϕ	=	$-Gm / r$
displacement of particle in s.h.m.	x	=	$x_o \sin \omega t$
velocity of particle in s.h.m.	v	=	$v_o \cos \omega t$
		=	$\pm \omega\sqrt{(x_o^{\,2} - x^2)}$
resistors in series,	R	=	$R_1 + R_2 + \ldots$
resistors in parallel,	$1/R$	=	$1/R_1 + 1/R_2 + \ldots$
electric potential,	V	=	$Q / 4\pi\varepsilon_o r$
alternating current/voltage,	x	=	$x_o \sin \omega t$
transmission coefficient,	T	=	exp(-2kd)
	where k	=	$\sqrt{\dfrac{8\pi^2 m(U-E)}{h^2}}$
radioactive decay,	x	=	$x_o \exp(-\lambda t)$
decay constant	λ	=	$\dfrac{0.693}{t_{\frac{1}{2}}}$

For Examiner's Use

Section A

Answer **all** the questions in this section.

1 **(a)** **(i)** Define *gravitational potential*. [2]

The gravitational potential at a point in the gravitational field is defined as the work done per unit mass to move a test mass from infinity to the point by an external agent (without change in kinetic energy).

(ii) Explain why gravitational potential has a negative value. [2]

The gravitational potential is always negative as gravitational field is an attractive field. The work done per unit mass to move a test mass from infinity to the point by an external agent will be negative since the force by the external agent is opposite in direction to the displacement of the test mass as it moves from infinity to a point in the gravitational field.

Since we defined the gravitational potential at infinity to be zero, all gravitational potential in the gravitational field must have a negative value..

(b) Values for the gravitational potential due to the Earth are given in the table below:

Distance from the Earth's surface/ m	Gravitational potential/ MJ kg^{-1}
0	-62.72
390 000	-59.12
400 000	-59.03
410 000	-58.94
infinity	0

(i) Calculate the loss in the potential energy of a satellite of mass 700 kg if it falls from a height of 400 000 m to the surface of the Earth. [2]

Loss in potential energy $= -m\Delta\phi$

$= -700[(-62.72)-(-59.03)]\times10^{6}$

$= 2.58\times10^{9}$ J

(ii) Deduce a value for the gravitational field strength of the Earth at a height of 400 000m. [2]

Since $g = -\dfrac{d\phi}{dr}$, to deduce g at 400000 m,

calculate the gradient of it using values at 390000 m and 410000 m

$$|g| = \left|\frac{\Delta\phi}{\Delta r}\right| = \left|\frac{[(-59.12)-(-58.94)]\times10^{6}}{390000-410000}\right| = 9.0 \text{ N kg}^{-1}$$

For Examiner's Use

2 **(a)** State the First Law of Thermodynamics. [1]

The First Law of Thermodynamics states that the **increase in internal energy** of a system is equal to the sum of the **heat supplied to** the system and the **work done on** the system.

Fig. 2.1 shows the variation with volume of the pressure of an ideal gas. The gas which is initially at state X, can be compressed to state Z either directly along the curve path XZ or indirectly from X to Y to Z.

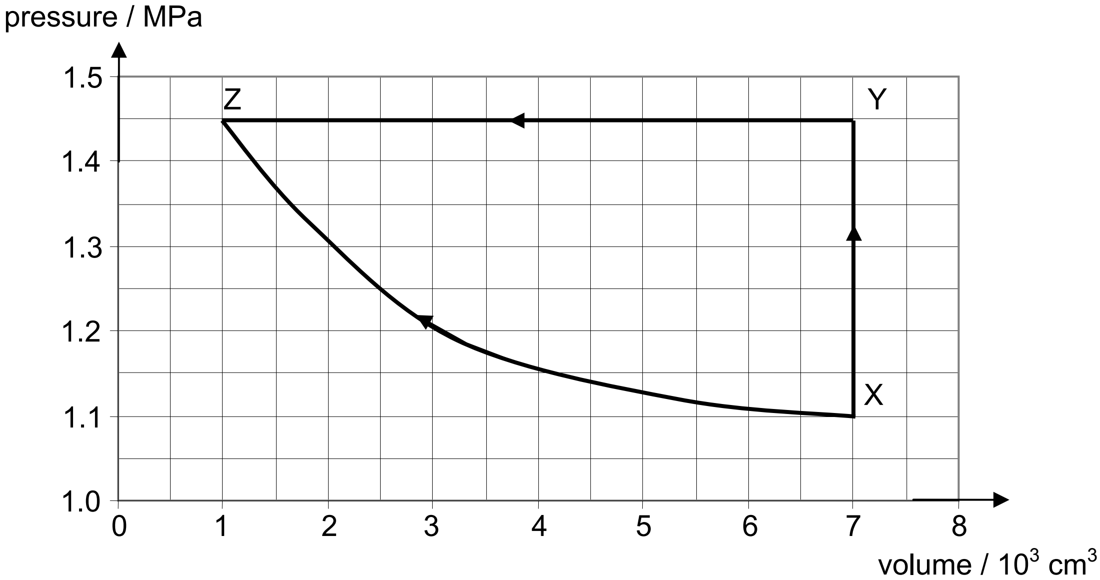


Fig. 2.1

(b) When the gas is compressed from X to Z along the curved path, 9000 J of heat energy is released to the surrounding.

(i) Using Fig. 2.1, estimate the work done on the gas. [2]

Work done for each square = $(0.05 \times 10^6)(0.5 \times 10^{-3}) = 25$ J

Estimated number of squares under the curve = 22 + 22 x 12

= 22 +264 = 286

$W_{XZ} = 25 \times 286 = 7150$ J = 7200 J (accept between 7100 to 7200)

(ii) Hence, calculate the change in internal energy of the gas. [1]

$\Delta U = W + Q$

$\Delta U = 7200 - 9000$

$\Delta U = -1800$ J

(c) When the gas is compressed from **X** to **Z** along the paths **XY** and **YZ**,

(i) determine the quantity of heat supplied to the gas. [2]

W_{XYZ}= Area under XYZ = $(1.45 \times 10^6)[(7.0-1.0) \times 10^{-3}] = 8700$ J

$\Delta U_{XYZ} = Q_{XYZ} + W_{XYZ}$

$Q_{XYZ} = -1800 - 8700 = -10500$ J

(ii) using the graph, explain whether the process from **Y** to **Z** is isothermal. [2]

From Y to Z, pressure is constant and volume decreases, hence the product of pV is not constant (decreases). Using pV = nRT, for the same amount of substance, T is not constant (decreases). Therefore process is not isothermal.

3 **(a)** Define *electric field strength* at a point in an electric field. [1]

The electric field strength at a point in an electric field is defined as the electrostatic force acting per unit positive charge placed at that point.

(b) Fig. 3.1 shows two isolated point charges **X** and **Y**. **X** carries a charge of +3.2 × 10^{-10} C, while **Y** carries a charge of -6.2 × 10^{-10} C. The two charges are 0.20 cm apart.

Sketch on Fig. 3.1, the resultant electric field lines due to the two charges. [3]

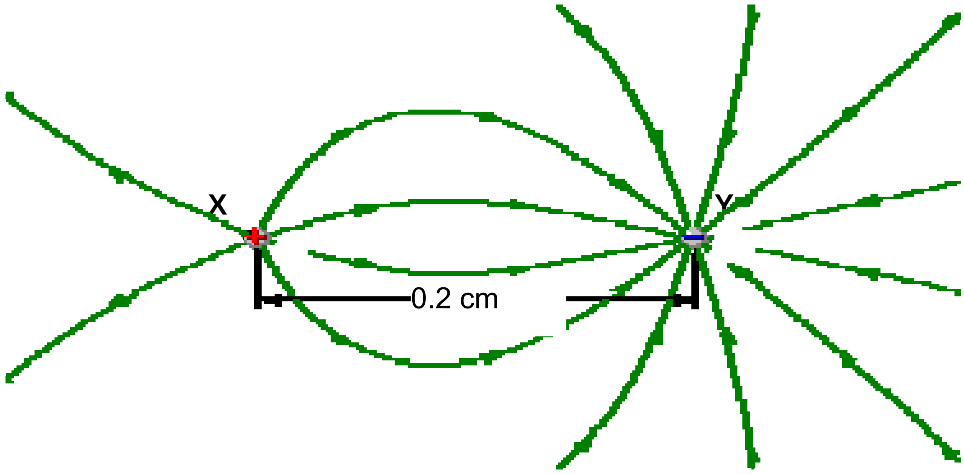


Fig. 3.1

Main features of drawing:

1. Arrows point from positive charge X to negative charge Y.
2. More field lines (nearly double) point towards the negative charge compared to the positive charge.
3. Asymmetry of the field lines between the two charges, with the 'hump' closer to the positive charge. Symmetry about the horizontal line joining the two charges.

(c) A test charge **T** of +1.2 × 10^{-10} C is placed 0.10 cm from **X** such that **XTY** forms a right angle as shown in Fig. 3.2.

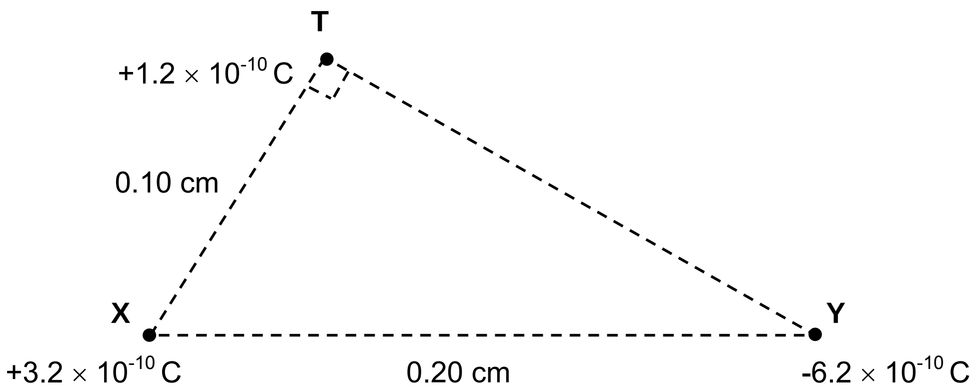


Fig. 3.2

(i) Calculate the magnitude of the net force, ***F***, acting on **T** and [3]

Distance TY = $\sqrt{0.20^2 - 0.10^2}$

= $\sqrt{0.03}$ cm

= 0.173 cm

Force on T due to Y,

$F_Y = \dfrac{q_Y q_T}{4\pi\varepsilon_o r^2}$

$= \dfrac{(6.2\times10^{-10})(1.2\times10^{-10})}{4\pi\varepsilon_o(\sqrt{0.03}\times10^{-2})^2}$

$= 2.23 \times 10^{-4}$ N

Force on T due to X,

$F_X = \dfrac{q_X q_T}{4\pi\varepsilon_o r^2}$

$= \dfrac{(3.2\times10^{-10})(1.2\times10^{-10})}{4\pi\varepsilon_o(0.10\times10^{-2})^2}$

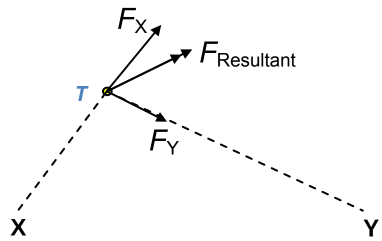
$= 3.45 \times 10^{-4}$ N

Magnitude of the net force acting on T, F

$= (\sqrt{3.45^2 + 2.23^2})\times10^{-4}$

$= 4.11 \times 10^{-4}$ N

(ii) indicate its general direction in Fig. 3.2 (Exact calculation for the direction is not required.) [1]



Direction of the net force on T is acting towards the top right hand corner.
[This must be indicated on the Figure as indicated in the instructions.]

4 A potential divider circuit consists of two resistors of resistances *A* and *B*, as shown in Fig. 4.1.

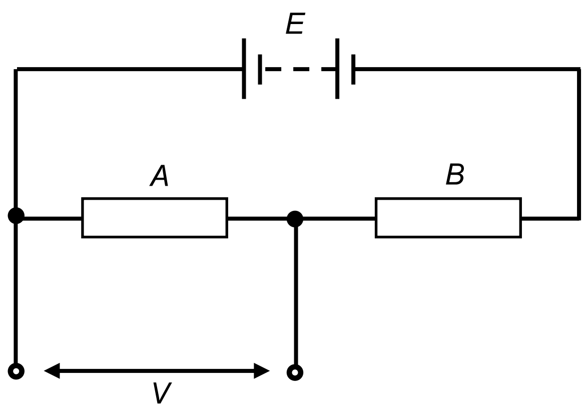


Fig. 4.1

The battery has e.m.f. *E* and negligible internal resistance.

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(a) Deduce that the potential difference V across the resistor of resistance A is given by the expression

$$V = \frac{A}{A+B}E$$

[2]

Current in series circuit $= \frac{E}{R_{tot}}$

$= \frac{E}{A+B}$

Potential difference across A $= IA$

$= \frac{E}{A+B} \times A$

$= \frac{A}{A+B}E$ [shown]

(b) The resistances A and B are 1500 Ω and 4000 Ω respectively.
A voltmeter is connected in parallel with the 1500 Ω resistor and a thermistor is connected in parallel with the 4000 Ω resistor, as shown in Fig. 4.2.

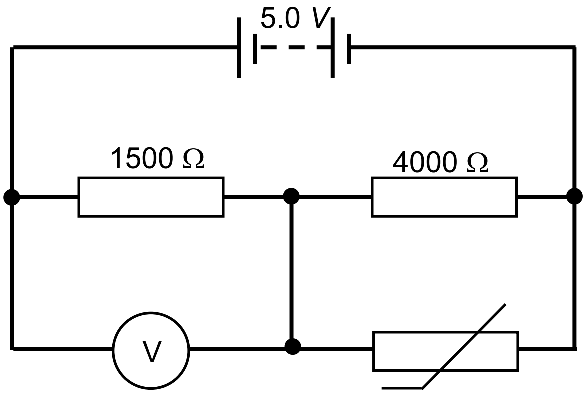

Fig. 4.2

The battery has e.m.f. 5.0 V. The voltmeter is ideal.

(i) State and explain qualitatively the change in the reading of the voltmeter as the temperature of the thermistor is raised. [3]

As the temperature of the thermistor is raised, its resistance will decrease. This will result in the combined resistance of the thermistor and the 4000 Ω resistance to decrease due to their parallel circuit arrangement. As a result, based on the potential divider principle, the potential across the 1500 Ω resistance will increase and hence, the voltmeter reading will increase.

For Examiner's Use

(ii) The voltmeter reads 2.4 V when the temperature of the thermistor is 20 °C. Calculate the resistance of the thermistor at 20 °C. [3]

Potential diff across 1500 Ω $= \dfrac{A}{A+R_{//}} \times E$

$$2.4 = \frac{1500}{1500+R_{//}} \times 5.0$$

$$R_{//} = 1625\ \Omega$$

$$\frac{1}{R_{//}} = \frac{1}{B} + \frac{1}{R_{thermistor}}$$

$$\frac{1}{1625} = \frac{1}{4000} + \frac{1}{R_{thermistor}}$$

$$R_{thermistor} = 2737$$

$$= 2.7 \times 10^3\ \Omega$$

5 Fig. 5.1 shows the variation with nucleon number of the binding energy per nucleon of a nucleus.

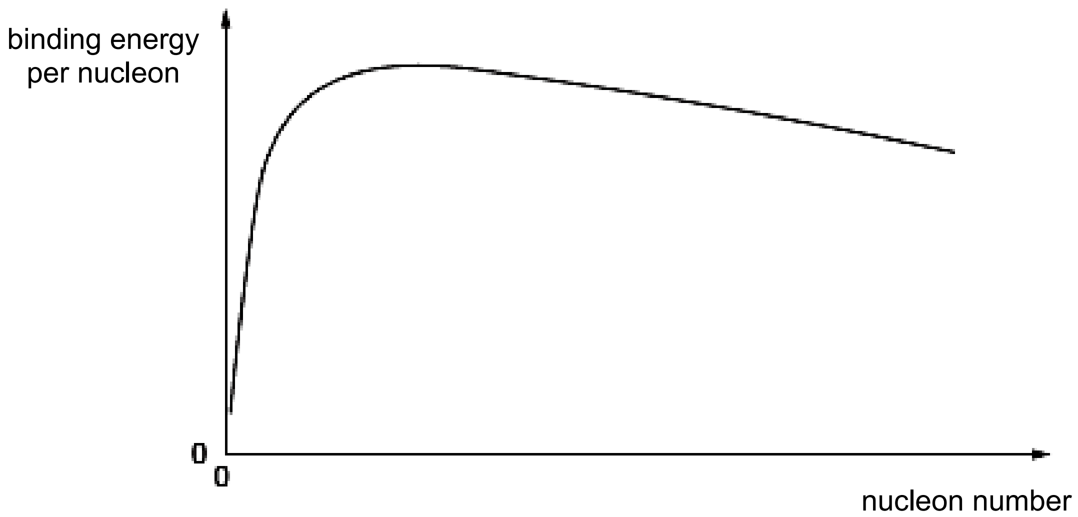

Fig. 5.1

(a) On Fig. 5.1, mark with the letter S the position of the nucleus with the greatest stability. [1]

S shown at the peak

(b) One possible fission reaction is

$${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{144}_{56}\text{Ba} + {}^{90}_{36}\text{Kr} + 2{}^{1}_{0}\text{n}$$

(i) On Fig. 5.1, mark possible positions for

1 the Uranium-235 nucleus and label this position U, [1]

2 the Krypton-90 nucleus and label this position Kr. [1]

Kr and U on right of peak in correct relative positions

For Examiner's Use

(ii) The binding energy per nucleon of each nucleus is as follows:

$^{235}_{92}$U: 1.2191×10^{-12} J

$^{144}_{56}$Ba: 1.3341×10^{-12} J

$^{90}_{36}$Kr: 1.3864×10^{-12} J

Use these data to calculate

1 the energy released in this fission reaction, giving your answer to three significant figures. [3]

binding energy of U-235 = 2.8649×10^{-10} J
binding energy of Ba-144 = 1.9211×10^{-10} J
binding energy of Kr-90 = 1.2478×10^{-10} J
energy release = 3.04×10^{-11} J

2 the mass equivalent of this energy. [1]

$E = mc^2 =$ 3.38×10^{-28} kg

(iii) Suggest why the neutrons were not considered in your calculation in **(ii)**. [1]

Neutrons are single particles which have no binding energy per nucleon

Section B

Answer **two** questions in this section.

6 **(a)** A structure consists of a sphere of mass 0.500 kg, attached firmly to one end of a light rod. The other end of the rod is pivoted freely at point O. The distance between the centre of gravity of the sphere to O is 0.400 m. The structure is held in a position such that the rod is at an angle of 5.00° from the vertical, as shown in Fig. 6.1.

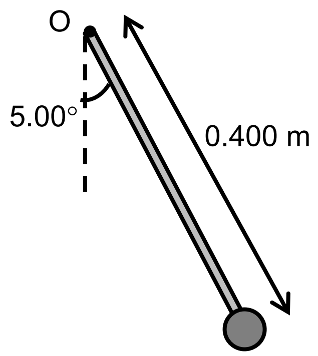

Fig. 6.1

The structure is then released from rest and oscillates in simple harmonic motion. At one instant during the oscillation, the sphere is directly below O, as shown in Fig. 6.2.

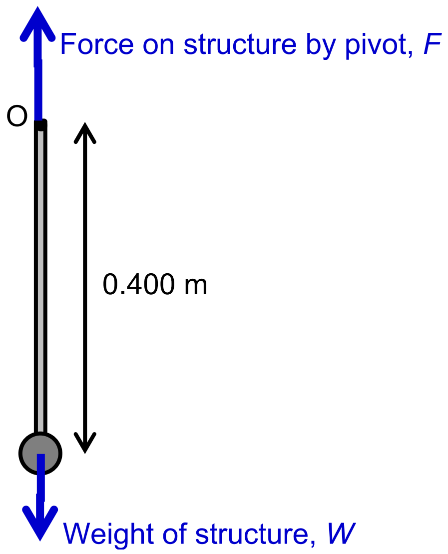

Fig. 6.2

(i) On Fig. 6.2, indicate the forces acting on the structure. [2]

(ii) Show that the linear speed of the sphere at the instant in Fig. 6.2 is 0.173 m s^{-1}. [2]

By conservation of energy,

Loss in GPE = Gain in KE

$mg(r - r\cos\theta) = 0.5mv^2$

$9.81(0.400)(1 - \cos 5.00) = 0.5v^2$

$v = 0.1728 = 0.173 \text{ m s}^{-1}$

(iii) Determine the force exerted on the pivot by the structure in Fig. 6.2. [3]

Since the structure is moving in circular motion,

by considering the structure,

$\sum F_c = ma_c$

$F - W = m\dfrac{v^2}{r}$

$F = 0.500(\dfrac{0.1728^2}{0.400} + 9.81)$

$F = 4.94 \text{ N}$

By Newton's 3rd Law, the force exerted on the structure by the pivot is equal in magnitude and opposite in direction to the force exerted on the pivot by the structure. [1] Hence force on pivot is 4.94 N.

(iv) With the aid of a diagram, discuss whether the force exerted by the pivot on the structure is *always upward* throughout the oscillation. [3]

Not always vertical. This is because as the structure is oscillating (at any instant between Fig. 6.1 and Fig. 6.2), there is a centripetal acceleration toward O, which is diagonal. This suggests that the resultant force needs to have a horizontal component. As weight of the structure is always vertically downward, the only force that can provide the horizontal component can only be from the pivot.

(b) A horizontal turntable is connected to a motor such that it rotates at exactly 47 revolutions per minute. A peg is fixed vertically on the turntable. A horizontal beam of light casts a shadow of the peg onto a screen in front of which is suspended the structure mentioned in **(a)**, as shown in Fig. 6.3.

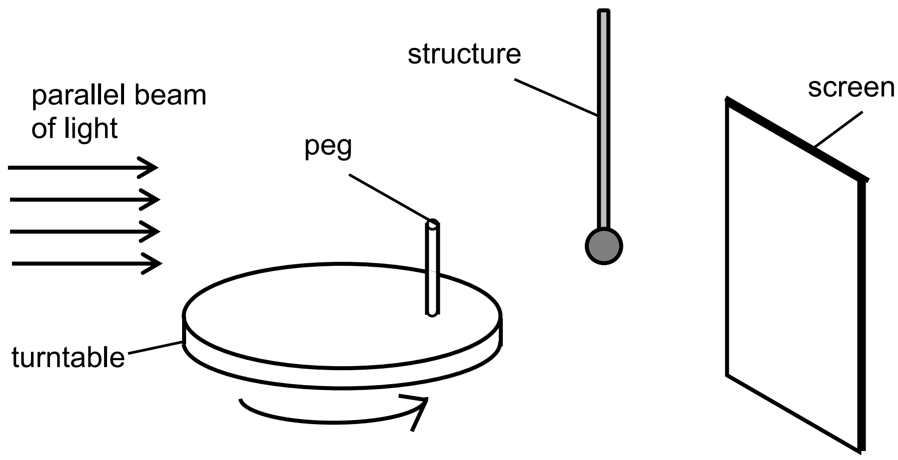

Fig. 6.3

The length of the structure is adjusted such that the shadows of the peg and the structure move in phase on the screen.

(i) Explain what is meant by *in phase* in this context. [1]

In phase means that the shadows of the peg and structure are always synchronized. The shadows always move to the same extreme location at the same time.

The angular speed of the turntable suddenly increases to $47\frac{1}{3}$ revolutions per minute.

(ii) Define *angular speed*. [1]

Angular velocity ω is the rate of change in angular displacement of a body.
Or
The rate of change in phase of an oscillator

(iii) Briefly describe what will be observed on the screen. [2]

The shadow of the peg will oscillate faster than that of the structure, and the two shadows will not be in phase anymore. The phase difference will increase with time.

(iv) Determine the time taken before the two shadows are next in phase. [2]

Since the peg is 1/3 revolution per minute faster than the structure, this means that the peg will cover 1/3 revolution more every one minute. To be in phase again, the peg needs to cover exactly one revolution more than the structure.

Hence time required $= 1 \div \frac{1}{3} = 3$ min = 180 s

(v) Calculate the number of oscillations made by the *peg* before the shadows are next in phase. [2]
Number of revolution = 3 × 47 + 1 = 142 revolutions

(vi) State two assumptions that were made in the calculations of **(b)(iv)** and **(b)(v).** [2]
There is no loss in energy of the structure throughout the motion and hence the period/angular speed of the structure remains constant.
The turntable is able to maintain its angular speed/period due to the motor.

7 (a) (i) State the *principle of superposition*. [1]
When two (or more) waves meet at (a point in space), the resultant displacement is the vector sum of the individual displacements

(ii) Explain what is meant when two sources are *coherent*. [1]
Waves emitted from the sources have a constant phase difference.

(b) Two sources S_1 and S_2 of sound are situated 80 cm apart in air, as shown in Fig. 7.1.

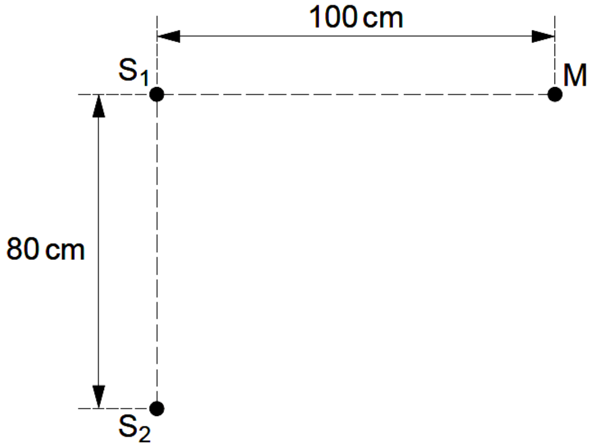

Fig. 7.1

The frequency of vibration can be varied. The two sources always vibrate in phase but have different amplitudes of vibration.

A microphone M is situated a distance 100 cm from S_1 along a line that is normal to S_1S_2.

As the frequency of S_1 and S_2 is gradually increased, the microphone M detects maxima and minima of intensity of sound.

(i) State the two conditions that must be satisfied for the intensity of sound at M to be zero. [2]
1. Waves from S_1 and S_2 arriving at M have a phase difference of 180° (or π rad).
or Waves from S_1 and S_2 arriving at M have a path difference that is an odd integral of half wavelength.

2. Waves from S_1 and S_2 arriving at M have the same amplitude (or intensity).
or Sources S_1 and S_2 have a ratio of amplitudes of 1.28.

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(ii) The speed of sound in air is 330 m s^{-1}.

The frequency of sound from S_1 and S_2 is increased. Determine the number of minima that will be detected at M as the frequency is increased from 1.0 kHz to 4.0 kHz. [4]

Path difference between waves from S_1 and S_2 is 28 cm.	
Wavelength of the sources changes from 33 cm to 8.25 cm.	$(2n+1)\pi = \frac{0.28}{\left(\frac{330}{f}\right)}(2\pi)$ For f = 1.0 kHz: n = 0.34 For f = 4.0 kHz: n = 2.9
Minima occurs when wavelength of the sources are (56 cm,) 18.7 cm, 11.2 cm, (8.0 cm, 6.22 cm …)	Since n can only be an integer, minima occurs when n = 1 & n = 2
Hence 2 minima observed.	

(iii) The variation with time of the displacement x of the sound waves arriving at M from S_1 and S_2 are as shown in Fig. 7.2a and Fig. 7.2b respectively.

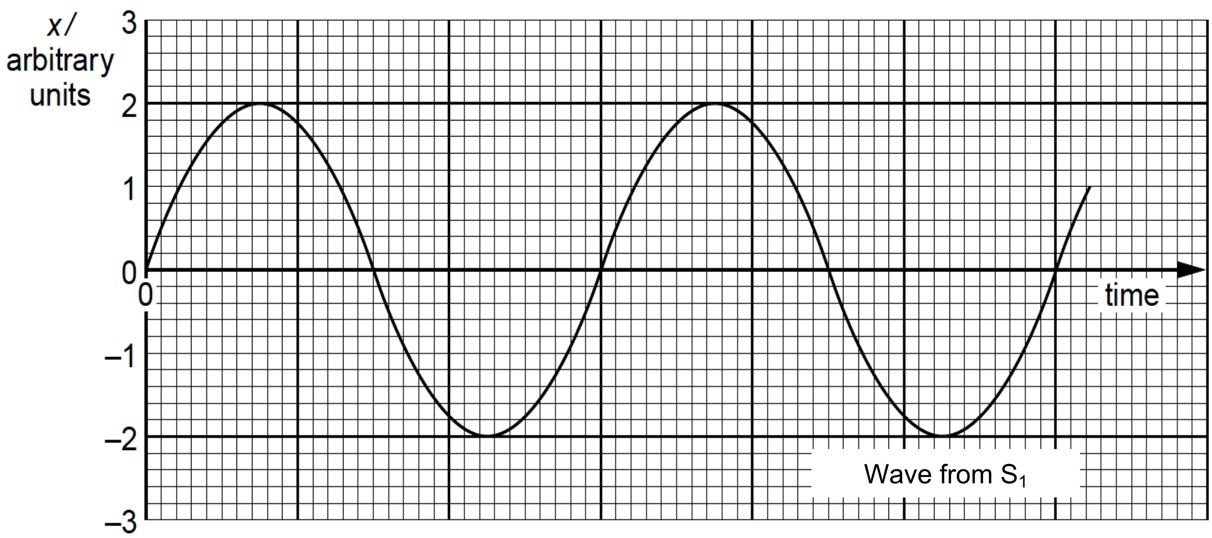

Fig. 7.2a

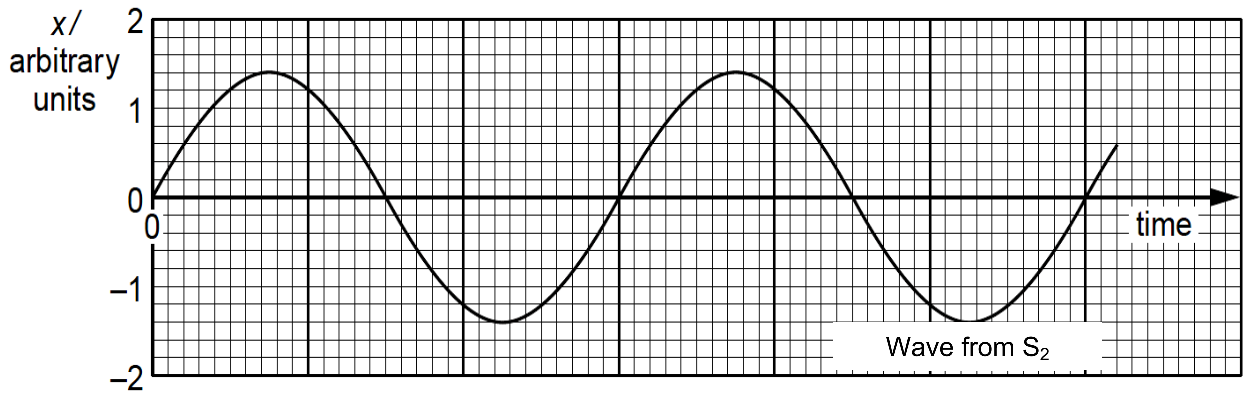

Fig. 7.2b

Determine the ratio of

$$\frac{\text{intensity of sound when a maxima is detected at M}}{\text{intensity of sound when a minima is detected at M}}$$

Amplitude of resultant wave at minima is 0.6 units and at maxima is 3.4 units

Intensity $\propto$ amplitude^2

$$\frac{\text{intensity when maxima is detected}}{\text{intensity when minima is detected}} = \left(\frac{3.4}{0.6}\right)^2 = 32$$

[3]

(c) Laser beam of red light of wavelength 644 nm is incident normally on a diffraction grating having 550 lines per millimetre, as illustrated in Fig. 7.3.

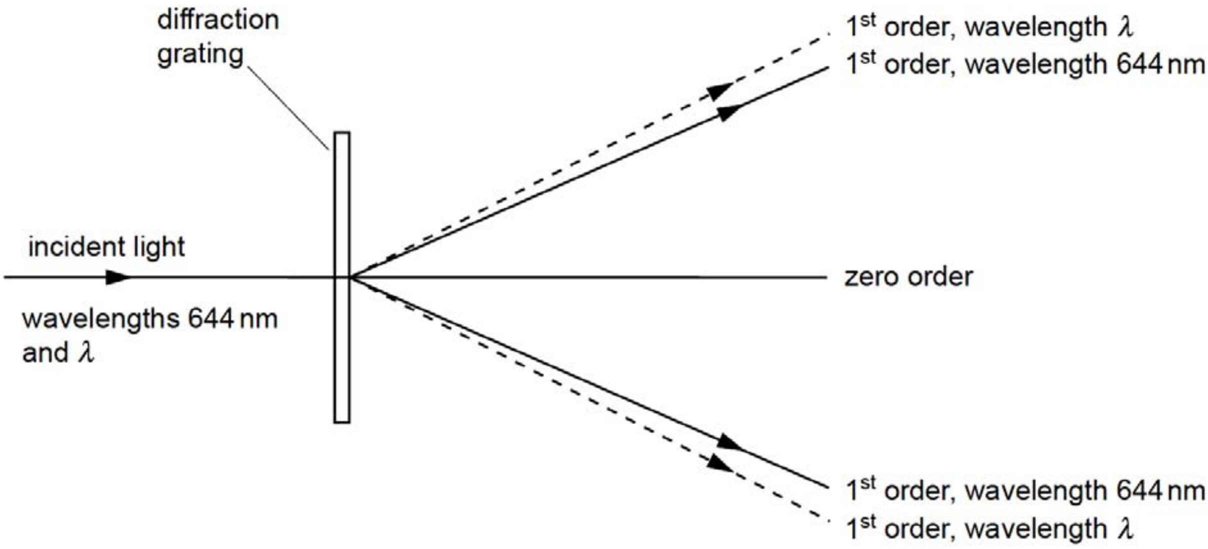

Fig. 7.3

Red light of wavelength λ is also incident normally on the grating. The first order diffracted light of both wavelengths is illustrated in Fig. 7.3.

(i) Determine the total number of bright spots of wavelength 644 nm that are visible. [3]

$d\sin\theta = n\lambda$

$\left(\frac{10^{-3}}{550}\right)\sin 90 = n(644 \times 10^{-9})$

$n = 2.8$

Highest order that can be seen is the 2nd order.

Hence total number of bright lines observed is 5.

(ii) State and explain

1 whether λ is greater or smaller than 644 nm, [1]

Since θ is greater, λ is also greater.

2 in which order of diffracted light there is the greatest separation of the two wavelengths. [2]

When n is larger, Δθ is larger, thus the greatest separation occurs in the second order.

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(iii) The diffraction grating is now rotated 90° about an axis parallel to the incident laser beam, as shown in Fig. 7.4.

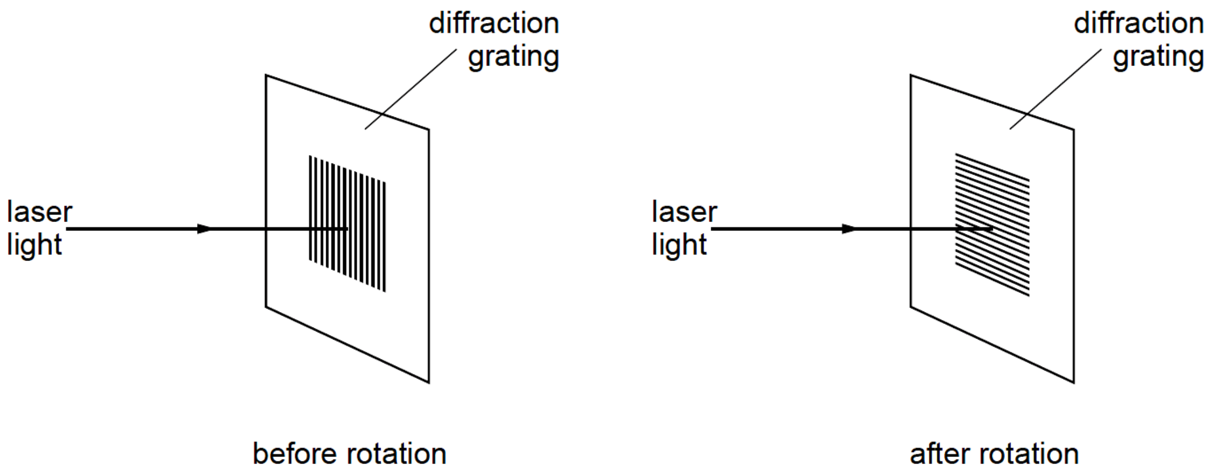

Fig. 7.4

State what effect, if any, this rotation will have on the diffraction pattern that is observed. [2]

0^{th} order (or central) maxima remains in the same position.

Diffraction pattern will rotate through 90^{o}

(iv) In another experiment using the same apparatus, a student notices that the angular separation between the zero order maxima and the two 1^{st} order maxima are not equal.

Suggest a reason for this difference. [1]

Grating is not normal to incident light

<u>**or**</u> Screen is not parallel to the grating

8 (a) 'X-rays are used to investigate the atomic structure of solids.' Deduce from this statement the wavelength of the X-rays used.

........The wavelength is in the order of the separation of the atoms. 10^{-10}m ....

.... [1]

(b) 'Sometimes, for example, in the case of rubber, electrons with a de Broglie wavelength of about 0.11 nm are used instead of X-rays.'
Determine the momentum of each electron.

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{0.11 \times 10^{-9}} = \mathbf{6.0 \times 10^{-24}} \text{ kg m s}^{-1}$$

momentum = ... N s [2]

(c) An X-ray tube operates with a potential difference of 100 kV between the anode and cathode. Fig. 8.1 is a sketch of the X-ray spectrum produced by this tube for a particular metal target. Fig. 8.2 shows a sketch of the energy level of target material and how the K_α line is formed. The tube voltage is 100 kV and the current is 20 mA.

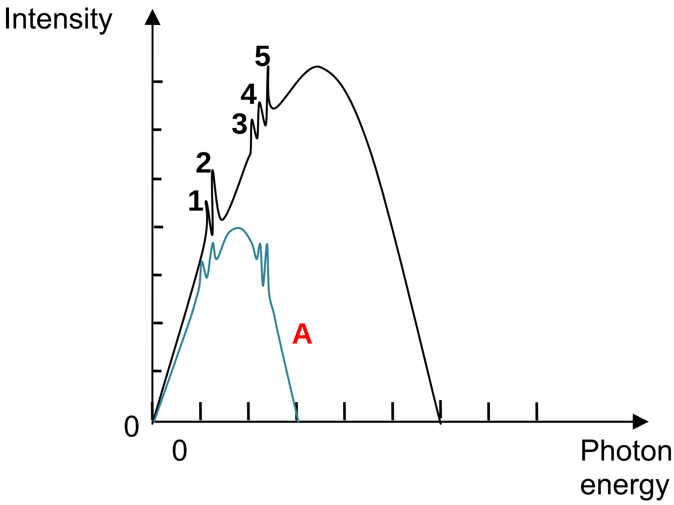

Fig. 8.1

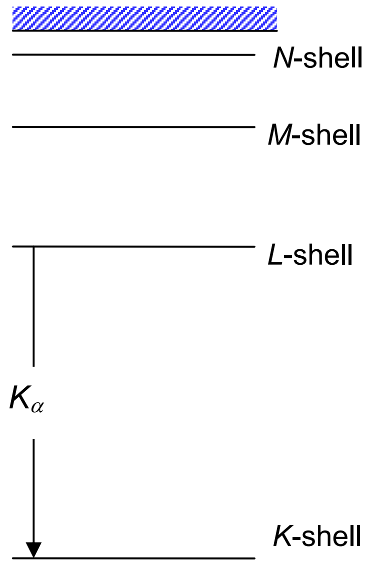

Fig. 8.2

(i) Calculate the maximum energy of an X-ray photon produced, explain your working.

An accelerated electron loses all its kinetic energy in one single collision when hitting the target and the energy is converted to only one photon. This most energetic photon is thus obtained.

$$hf_{max} = \frac{1}{2} mv^2 = eV = 1.60 \times 10^{-19} \times 100 \times 10^3 = 1.60 \times 10^{-14} \text{ J}$$

maximum energy = ... J [2]

(ii) With reference to Fig. 8.1, write down the numbers that represent the spectrum lines K_α and L_β.

K_α: 3 .... L_β: 2 .... [2]

(iii) Sketch on Fig. 8.1 a spectrum for X-ray from the tube if the tube voltage is reduced to 50 kV, the current at 10 mA. Label this spectrum A. [2]

(d) Explain how the characteristic and continuous parts of the spectrum are formed.

(i) Formation of characteristic parts of X-ray spectrum:

Line spectrum: When a high speed electron knocks out an orbiting electron in the inner shells of a target atom, a 'hole' is formed. When an electron from an outer shell of this atom fall into this 'hole', an X-ray photon of a particular frequency will be emitted.

.... [2]

For Examiner's Use

(ii) Formation of continuous parts of X-ray spectrum:

According to classical physics, when the electrons are accelerating towards the target and when they are decelerating when hitting the target, electromagnetic waves will be emitted. In this case, the EM waves emitted is of high energy and is in the X-rays region. Because the acceleration and deceleration are continuous processes, the energy converted to X-rays can be any values. Hence continuous spectrum is formed.

.... [2]

(e) The energy required to remove an electron from the various shells of the nickel atom is:

K shell 1.36×10^{-15} J
L shell 0.16×10^{-15} J
M shell 0.08×10^{-15} J

An X-ray tube with a nickel target emits the X-ray K_α radiation of nickel.

Determine

(i) the minimum potential difference across the tube,

To have a K_α line formed, the incident electrons must have enough energy to knock out an atom from K-shell to infinity.

$eV_{min} = 1.36 \times 10^{-15}$

$V_{min} = 1.36 \times 10^{-15}/1.60 \times 10^{-19} = 8490$ V

potential difference = ... V [2]

(ii) the energy of the X-ray quantum of longest wavelength in the K-spectrum of nickel.

The least energetic line is given by K_α, when electron from L-shell falls to fill the hole in the K-shell.

$h f_{min} = 1.36 \times 10^{-15} - 0.16 \times 10^{-15} = 1.20 \times 10^{-15}$ J

energy = ... J [2]

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(f) A beam of electrons moving in the x-axis in an X-ray tube with momentum 4×10^{-23} kg m s^{-1} in the x-axis passes through a 3 mm slit in an anode before it hits the target as shown in Fig. 8.3. The uncertainty of the y-position of the electrons can be considered in the order of the size of the slit. Use uncertainty principle to estimate the possible angular spread θ of the electron beam after passing through the slit.

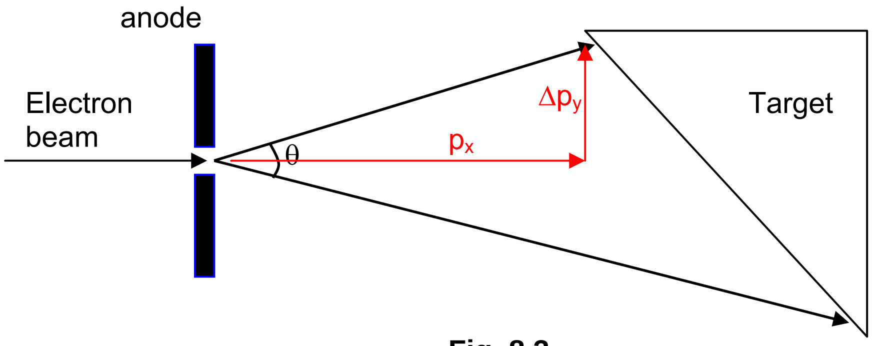

Fig. 8.3

$$\Delta p_y \Delta y \geq \frac{\hbar}{2}$$

$$\Delta p_y \text{ x } 3 \text{ x } 10^{-3} \geq \frac{6.63x10^{-34}}{4\pi}$$ [1]

$$\Delta p_y \text{x } 3 \text{ x } 10^{-3} \geq 5.27 \text{ x } 10^{-35}$$

$$\Delta p_y \geq 1.8 \text{ x } 10^{-32} \text{kg m s}^{-1}$$ [1]

$$\tan \frac{\theta}{2} = \frac{\Delta p_y}{p_x} = \frac{1.8 \text{ x } 10^{-32}}{4 \text{ x } 10^{-23}} = 4.5x10^{-10}$$

$$\theta = 4.5 \text{ x } 10^{-10} \text{x } 2 = 9 \text{ x } 10^{-10} \text{ rad}$$ [1]

θ = ... rad [3]