H2 Mathematics

A Levels Papers Answer Key (2007 – 2012)

2007 Paper 1

1. $x > 7$, $-2 < x < -1$, $x < -3$
2. (ii) $f^{-1}(x) = \dfrac{1+3x}{x}$, $x \neq 0$
3. (b) $-1 + 2i$
4. $I = \dfrac{1}{3}\left(2 + 4e^{-\frac{3}{4}t}\right)$
5. $A = 2$, $B = 3$; translate 2 units in the negative x-direction, scale by a factor of 3 parallel to the y-axis, translate 2 units in the positive y-direction
6. (ii) $\begin{pmatrix} 4/3 \\ 2/3 \\ 5/3 \end{pmatrix}$ (iii) $\sqrt{35}$
7. (i) $re^{-i\theta}$ (ii) $2e^{\frac{i(\pi+2k\pi)}{6}}$, $k = 0, \pm1, \pm2, -3$
   (iii) $(z^2+4)(z^2-2\sqrt{3}z+4)(z^2+2\sqrt{3}z+4)$
8. (i) (2.5, 1.5, 5.5) (ii) $78.8°$ (iii) 2.14
9. (i) 0.619, 1.512
10. (ii) $3a$ (iii) $6 \le x \le 13$
11. (iii) $\dfrac{2}{5}$

2007 Paper 2

1. \$7.65
2. (i) $1-\dfrac{1}{(N+1)^2}$ (iv) $1-\dfrac{1}{N^2}$
3. (i) $1+nx+\dfrac{n(n-1)}{2}x^2+\dfrac{n(n-1)(n-2)}{6}x^3+\ldots$ (ii) $8-3x+\dfrac{387}{16}x^2-\dfrac{1151}{128}x^3$
   (iii) $-\dfrac{1}{\sqrt{2}} < x < \dfrac{1}{\sqrt{2}}$
4. (i) $\dfrac{5\pi}{6}+\dfrac{\sqrt{3}}{8}$; $\dfrac{5\pi}{6}-\dfrac{\sqrt{3}}{8}$ (ii)(a) π - 2 (b) 5.391
5. –
6. 0.933 (i) 0.717 (ii) 0.616
7. 30.8, 33.7; $p = 0.038$, reject H_0
8. (i) 0.395 (ii) 0.160 (iii) 0.392
9. (i)(a) 479001600 (b) 46080 (ii)(a) 39916800 (b)86400 (c) 240
10. (i) $\dfrac{1}{64}$ (ii) $\dfrac{21}{256}$ (iii) $\dfrac{13}{17}$
11. $x = 66.2 - 0.260t$; -11.7 or -11.8 (i) -0.994 (ii) 155 min

2008 Paper 1

1. 2.73
2. –
3. (i) $\begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix}$ (ii) $87.8°$ (iii) $\sqrt{675}$
4. (i) $\frac{3}{2}\ln(x^2+1)+C$ (ii) $\frac{3}{2}\ln(x^2+1)+2$ (iii) 0
5. (i) $\frac{\pi}{9}$ (ii) $\frac{ne^{n+1}-1}{(n+1)^2}$
6. (a) $2+\frac{3}{4}\theta^2$ (b) 1, 4, 16; $1 + 4x + 8x^2$
7. $x = 6.09$, $y = 12.6$
8. (i) -8 (ii) -3, 6 (iii) $1\pm i\sqrt{3}$, $-\frac{1}{2}$
9. –
10. (i) 1st Oct 2011 (ii)(a) \$6.08 (b) \$310.30 (c) 81
11. $\left(-\frac{4}{11},-\frac{4}{11},\frac{7}{11}\right)$ (i) $\mathbf{r} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (ii) $\lambda = -22$, $\mu \neq 17$ (iv) $-3x + y + 2z = 2$

2008 Paper 2

1. (ii) $x + x^2 + \frac{1}{3}x^3$ (iv) $-1.96 < x < 1.56$
2. (i) 0.999 (ii) $\frac{4}{15}\pi$ (iii) $\frac{2}{3}$
3. (a) 1, 2θ, $\frac{1}{5}\pi$, $\frac{2}{5}\pi$ (b)(ii) 0.058, 1.229
4. (ii) $f^{-1}(x) = 4+\sqrt{x-1}$, $x > 1$ (iv) $\frac{1}{2}(9+\sqrt{13})$
5. –
6. p-value = 0.121, do not reject H_0
7. (i) 0.5 (ii) 0.512 (iii) 0.109
8. (i) 0.970 (ii) 1.42, 4.40 (v) 8.3
9. 0.109; 0.192; 0.134
10. (i) 120 (ii) 9 (iii) 210 (iv) 485
11. (i) 0.0385 (ii) 0.0925; 110, 576; $a = 3$, $b = -40$

2009 Paper 1

1. (i) $1.5n^2 - 8.5n + 17$ (ii) $n \geq 11$, for integers n
2. $p = \dfrac{2\pi}{2\ln 3}$
3. (i) $A = 2$ (ii) $\dfrac{1}{2(n+1)} - \dfrac{1}{2n} + \dfrac{1}{4}$ (iii) $\dfrac{1}{4}$
4. (i) 11 (iii) $\dfrac{110}{3}$
5. $\dfrac{1}{6}n(2n+1)(7n+1)$
6. (iii) -0.515, 2.45
7. (i) e, 0, $-e$; $e - \dfrac{1}{2}ex^2$ (ii) $a = \dfrac{1}{e}$, $b = \dfrac{1}{2e}$
8. (i) $S_\infty = 356.3 < 357$ (ii) L = 272.26; 10 (iii) a = 16.8, $d = \pm 0.491$
9. (i) $2^{\frac{1}{14}} e^{i\left(\frac{\pi}{28} + \frac{2k\pi}{7}\right)}$, k = 0, ± 1, ± 2, ± 3 (iii) $y = \left(\tan\dfrac{5\pi}{28}\right)x$ or $y = \left(\tan\dfrac{-23\pi}{28}\right)x$
10. (i) 70.9 (ii) $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$, $\lambda \in \mathbb{R}$ (iii) $x - y$ = -1
11. (ii) $\left(\pm\dfrac{1}{\sqrt{2}}, \pm\dfrac{1}{\sqrt{2}}e^{-\frac{1}{2}}\right)$ (iii) $\dfrac{1}{2} - \dfrac{1}{2}e^{-n^2}$; $\dfrac{1}{2}$ (iv) $1 - e^{-4}$ (v) 0.36

2009 Paper 2

1. (ii) $y = 2x - 12$ (iii) Q(-3, -18)
2. (i) (12, -4, 6) (iv) 138.6
3. (i) $\mathrm{f}^{-1} = \mathrm{f}$, range is all real values except $\dfrac{a}{b}$ or we write $y \in \mathbb{R}$, $y \neq \dfrac{a}{b}$ (iii) 0, $\dfrac{2a}{b}$
4. (ii) $n = 150 - 50e^{\frac{-t}{50}}$, n increases to a limit of 150 000
5. –
6. (ii) Inappropriate (iv) 3 min 41.4 s; unreliable
7. (i) 0.035
8. (i) 6720 (ii) 5040 (iii) 192 (iv) 480
9. (i) 25 (ii) 0.2035 (iii) 0.0707
10. (i) 9.6, 0.81 (ii) p = 0.219, do not reject H_0
11. (ii) 0.3464 (ii) 0.0391 (iv) 0.1517 (v) $(1-p)^{20} + 20p(1-p)^{19} = 0.2$; 0.142

2010 Paper 1

1. (i) $p = \frac{3}{7}$
2. (i) $1 + 3x + \frac{5}{2}x^2 + \ldots$ (ii) $n = \frac{9}{4}$
3. (i) $u_n = 4n - 2 + c$ (ii) $u_{n+1} = u_n + 4$
4. $\frac{dy}{dx} = \frac{x+y}{y-x}$ (ii) $(\sqrt{2}, -\sqrt{2})$ and $(-\sqrt{2}, \sqrt{2})$
5. (i) $y = \frac{(x-2)^3}{2} - 6$; (0, -10) and $(2 + \sqrt[3]{12}, 0)$
6. (i) $\beta = 0.347$, $\gamma = 1.532$ (ii) 0.781 (iii) $\frac{9}{4}$ (iv) $-1 < k < 3$
7. 6.93; For large t, θ approaches 20
8. (i) $z_1 = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$; $z_2 = \sqrt{2}\left[\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right]$

   (ii) $\left(\frac{z_1}{z_2}\right)^* = \sqrt{2}(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12})$ (iv) $x = 2$
9. (i) $x = \sqrt[3]{\frac{200}{3}(k+1)}$ (ii) $\frac{y}{x} = \frac{3}{2(k+1)}$ (iii) $\frac{3}{4} \le \frac{y}{x} < \frac{3}{2}$ (iv) $k = \frac{1}{2}$
10. (ii) $(\frac{17}{2}, 2, \frac{3}{2})$ (iii) $B = (19, -19, -30)$ (iv) 348
11. (iii) $x^2 - y^2 = 4$

2010 Paper 2

1. (i) $x = 3 \pm 5i$ (ii) a = -16, b = -20; -2, -i, 2
2. (ii)(b) $\frac{3}{4}$
3. (i) $\frac{dy}{dx} = \frac{3x+4}{2\sqrt{x+2}}$; $x = -\frac{4}{3}$ (ii)(a) $\pm\sqrt{2}$
4. (ii) If the domain of f is restricted to $x \ge 0$, then any horizontal line will cut the graph at most once, so f is one-one and f^{-1} exists. Hence the least value of k is 0.
   (iv) $2 < x < 4$, $x \ne 3$ (v) $(-\infty, -1) \cup (0, \infty)$
5. –
6. $\bar{x} = 41.3$, $s^2 = 1.584$; p-value 0.0949, reject H_0
7. (i) 0.32 (ii) 0.92 (iii) 0.457 (iv) 0.15 (v) $P(A' \cap B \cap C) \le 0.15$
8. (i) $\frac{3}{5}$ (ii) $\frac{1}{10}$ (iii) $\frac{7}{20}$
9. (i) 0.681 (ii) 0.234 (iii) 0.362
10. (ii)(a) r = 0.9860 (b) r = 0.9907 (iii) $F = c + dv^2$
    (iv) $F = 0.024242v^2 + 3.1957$; v = 30.7
11. (i) 0.0655 (ii) t = 32 (iii) 0.192 (iv) 0.235 (v) 0.959

2011 Paper 1

1. $-2 < x < 1$
2. (i) $a = 0.215$, $b = -0.490$, $c = 3.281$ (ii) $\{x \in \mathbb{R} : x > 1.14\}$
3. (i) $y = \frac{-1}{p^3}x + \frac{3}{p}$ (ii) $Q(3p^2, 0)$, $P\left(0, \frac{3}{p}\right)$ (iii) $y^2 = \frac{27}{8x}$
4. (i) $1 - 3x^2 + 4x^4 + \ldots$ (ii)(a) 0.540 (b) 0.475
5. (ii) $\{x \in \mathbb{R} : 0 \le x \le 2\}$ (iii) $a = 2 + \sqrt{5}$
6. (ii) $\sum_{r=1}^{n} \cos r\theta = \frac{1}{2\sin\frac{1}{2}\theta}\left(\sin\left(n + \frac{1}{2}\right)\theta - \sin\frac{1}{2}\theta\right)$
7. (i) $\overrightarrow{OM} = \frac{1}{6}\mathbf{a} + \frac{3}{10}\mathbf{b}$; $k = \frac{1}{20}$ (ii)(a) $p = \frac{1}{7}$ (b) Length of projection of **b** on **a**
   (c) $\frac{1}{7}\begin{pmatrix} 9 \\ 7 \\ 8 \end{pmatrix}$
8. (i) $\frac{1}{20}\ln\left|\frac{10+v}{10-v}\right| + c$ (ii)(a) $\frac{10+v}{10-v} = Ae^{2t}$; $t = \frac{1}{2}\ln 3$ seconds (b) $\frac{10(e^2-1)}{e^2+1}$
   (c) Speed of stone for large values of t tends towards 10 metres per second.
9. (i) 4810 (ii) 40
10. (i) $z_1 = 2 - 2i$; $z_2 = -2 + 2i$ (ii) $w_1 = -1 - i$; $w_2 = -3 + i$
    (iv) These two loci happen to be parallel to each other (both gradients are 1)
11. (i) $x + y + 2z = -3$ (ii) $k = -7$ (iii) (-1, 6, -4) (iv) 22.2°

2011 Paper 2

1. (ii) Minimum $|z| = \sqrt{29} - 3$; Maximum $|z| = \sqrt{29} + 3$ (iii) Maximum $|z - 6 - i| = \sqrt{17}$
2. (ii) $\left(\frac{1}{2} - \frac{\sqrt{3}}{6}\right)n$; From the diagram, we deduce that $2x < n$, and this $x = \left(\frac{1}{2} - \frac{\sqrt{3}}{6}\right)n$ is the only answer
3. (i) $\mathrm{f}^{-1}(x) = \frac{1}{2}(e^{x-3} - 1)$; $\mathrm{D}_{\mathrm{f}^{-1}} = (-\infty, \infty)$, $\mathrm{R}_{\mathrm{f}^{-1}} = \left(-\frac{1}{2}, \infty\right)$
   (ii) $x = 0.4847$ and 5.482
4. (a)(i) $\int_0^n x^2 e^{-2x}\mathrm{d}x = \frac{-e^{-2n}}{4}(2n^2 + 2n + 1) + \frac{1}{4}$ (ii) $\frac{1}{4}$ (b) $2\pi(\pi - 2)$
5. $\mu = 53.7$, $\sigma = 8.32$
6. –
7. (iii) 0.552 (iv) 0.114
8. (ii) $r = 0.992$ (iv) $y = -0.85621x^2 + 22.23049$; 13.5
9. (i)(a) 0.058 (b) 0.517 (ii)(a) 0.109272 (b) 0.313

10. (i) H_0: $\mu = 38.0$, H_1: $\mu < 38.0$ (ii) $\left\{\bar{t} \in \mathbb{R} : 0 < \bar{t} < 36.8\right\}$

(iii) $\mathrm{P}\left(Z \le \dfrac{37.1-38.0}{\frac{5.0}{\sqrt{n}}}\right) > 0.05$; $\left\{n \in \mathbb{Z} : 1 \le n \le 83\right\}$

11. (i) 0.0941 (ii) 6

12. (i) 0.113 (ii) $e^{-0.02t} + 0.02te^{-0.02t} = 0.7$; t = 55 seconds (iii) 0.645

(iv) If the time period is in terms of several hours, the rate of people joining the check-in queue would probably not be constant throughout.

2012 Paper 1

1. Under 16, \$7.65; Between 16 & 65, \$9.85; Over 65, \$8.52
2. (i) $\frac{1}{4}\ln(1+x^4)+C$ (ii) $\frac{1}{2}\tan^{-1}x^2+C$ (iii) 0.186
3. (i) $u_2 = \frac{5}{6}$, $u_3 = \frac{1}{4}$ (ii) $l = -\frac{1}{3}$
4. (ii) $a = 1$, $b = \frac{3}{2}$
5. (i) $\mu = 6$ (ii) C = (5, 7, 1), $(\frac{17}{3}, \frac{19}{3}, \frac{5}{3})$
6. (i) $z^3 = 1-3c^2+i(3c-c^3)$ (ii) $z = 1 \pm i\sqrt{3}$ (iii) n = 10; $\left|z^{10}\right| = 1024$, $\arg z^{10} = \frac{2\pi}{3}$
7. (iii) $y = x$; Translate by 1 unit in direction of positive x-axis, scale by factor (k + 1) parallel to y-axis, translate by 1 unit in direction of positive y-axis
8. (iii) When $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -1 < 0$, hence turning point is a maximum
9. (i) $\mathbf{r} = \begin{pmatrix}7\\8\\9\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\1\end{pmatrix}, \lambda \in \mathbb{R}$ (ii) $\overrightarrow{ON} = \begin{pmatrix}5\\4\\7\end{pmatrix}$; $AN : NB = 1 : 3$ (iii) $x-7 = \frac{y-8}{-4} = z-9$
10. (i) $r = \sqrt[3]{\frac{3k}{5\pi}}$, $h = \sqrt[3]{\frac{3k}{5\pi}}$ (ii) r = 3.04, h = 4.88
11. (i) The tangents become parallel to the y-axis (iii) 3π

2012 Paper 2

1. (a) $y = 8x^2 - \frac{3}{4}x^4 + Cx + D$ (b) $t = \frac{1}{24}\ln\left|\frac{4+3u}{7(4-3u)}\right|$
2. (ii)(a) $\sqrt{58} - 4$ (b) $7 - \frac{28}{\sqrt{58}} - i(3 - \frac{12}{\sqrt{58}})$ (iii) -0.9579
3. (ii) $x = 2$ (iii) $x = -1$ (v) $x^3 + x^2 - 2x - 4 = 4$ and $-x^3 - x^2 + 2x + 4 = 4$; x = -2, 0, 1, 2
4. (i) 1 December 2002 (ii) $20100(1.005^n - 1)$; 45th month i.e. September 2004
   (iii) 1.8%
5. (i)(a) 0.00599 (b) 0.166 (ii) p = 0.999666
6. (i) H_0: $\mu = 14$, H_1: $\mu \neq 14$ (ii) $12.3 < \bar{x} < 15.7$ (iii) H_0 will be rejected
7. (i) $\frac{2}{15}$ (ii) $\frac{34}{35}$ (iii) $\frac{2}{455}$ (iv) $\frac{43}{273}$ (v) $\frac{1}{7}$
8. (ii) The practice paper may be more difficult than the other papers.
   (iv) L = 91, r = -0.929744 (v) L = 92 (vi) 13th week
   (vii) L is the percentage mark she will get if she continues practising indefinitely.
9. (i) The probability that a voter supports the Alliance Party is constant and independent of the other voters.
   (ii) 0.373 (iii)(a) Possible (b) Not possible (iv) p = 0.39
10. (i) Gold coins are scattered randomly per square metre and independently of each other. The mean number of coins per square metre is constant.
    (ii) 0.0474 (iii) $e^{-0.8x}0.8x = 0.2$; x = 0.324 (iv) 0.144 (v) 0.245 (vi) 0.912