

Section A: Pure Mathematics [35 marks]

- 1 Find the exact coordinates of the point(s) of intersection between the curve

$$y = \frac{x+2}{1-x} \text{ and the line } y = 2x+3.$$

[4]

- 2 Find the range of values of x for which $2x^2 - x - 1 < 0$.

Hence solve the inequality $2e^x - \frac{1}{e^x} < 1$.

[4]

- 3 (i) Sketch the curve whose equation is $y = 2 + \ln(x+3)$ indicating clearly the asymptotes, if any, and the exact axial intercepts.

[3]

- (ii) Find the exact coordinates of the point P on the curve at which the gradient of the tangent is $\frac{1}{3}$.

[2]

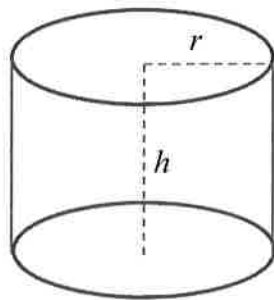
- (iii) The tangent to the curve at the point P cuts the x -axis at A and the normal at the point P cuts the x -axis at B . Calculate exact area of the triangle ABP .

[5]

- 4 (a) Find the set of values of x for which the curve $y = x^3 - 4x + 5$ is increasing, giving your answer in exact form.

[2]

- (b) A closed cylindrical jar has a fixed surface area of $100\pi \text{ cm}^2$. The height and the base radius of the jar are $h \text{ cm}$ and $r \text{ cm}$ respectively.



- (i) Show that the volume, $V \text{ cm}^3$, of the jar is $V = \pi r(50 - r^2)$.

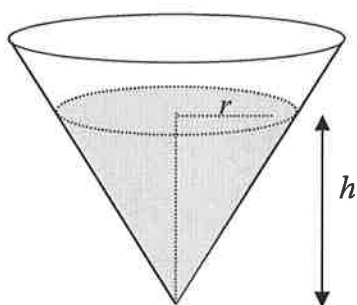
[3]

- (ii) Find the value of r , for which the volume of the jar is maximum.

[2]

- (c) Water is poured into a small conical vessel at a rate of $10 \text{ cm}^3/\text{sec}$. Given that the height of the water is twice as much as the radius of the surface area of the water at any instance, find the exact rate of increase in radius of the surface of the water when the height is 4 cm.

[Volume of a cone with radius r and height h is given as $\frac{1}{3}\pi r^2 h$]



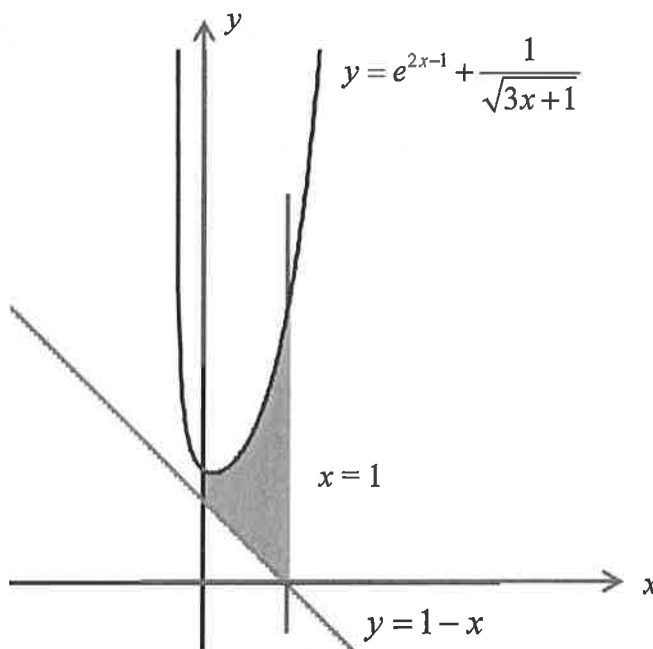
[3]

- 5(a) Show that $\frac{d}{dx}(\sqrt{2x^2 - x^3}) = \frac{4-3x}{2\sqrt{2-x}}$, given that $0 < x < 2$

Hence evaluate $\int \frac{4-3x}{\sqrt{2-x}} dx$

[3]

- (b)



Find the exact area of the shaded region, in the above figure, which is bounded by the curve $y = e^{2x-1} + \frac{1}{\sqrt{3x+1}}$, the y -axis, the line $y = 1-x$ and the line $x = 1$

[4]

[Turn Over]

Section B: Statistics [60 marks]

6 A machine grades mangoes according to their mass. Mangoes with a mass exceeding 385g are rejected as too large and mangoes with mass less than 325g are rejected as too small. A large batch of mangoes is graded and it is found that 13% are rejected as too large and 11% are rejected as too small. Assuming a normal distribution, find the mean mass and the variance of a randomly chosen mango from the batch. [4]

7(a) A professor is studying the sleep patterns of students at her university. There are 1800 male students and 1200 female students in that university. She decides to start by asking a random sample of 50 students how many hours of sleep they get on weekday nights.

The professor assigns each student a unique number from 1 to 3000. She selects the sample by randomly choosing one of the first 60 numbers and every 60th number thereafter.

(i) State one disadvantage of this sampling method in this case. [1]

(ii) Suggest a better sampling method and describe how the professor can choose a random sample of 50 students using this method. [2]

(b) A sample of 100 college students was chosen and the students were given a test to measure their IQ. Following is the frequency table which consists of the students' IQ and the corresponding number of students.

IQ (x)	97	98	99	100	101	102	103	104	105	106	107	108
No. of Students (f)	5	7	12	13	15	11	10	8	6	7	3	3

(i) Find the unbiased estimates of the population mean and population variance of distribution of IQ among college students. [2]

(ii) Using the estimated mean and variance obtained in part **(i)**, find the probability that the mean IQ of 60 randomly chosen college students lies between 98 and 101. [2]

- 8 On average 5% of computer chips manufactured in a firm are defective. A sample of twenty such computer chips is chosen randomly from the firm.
- (i) Evaluate the mean and variance of the number of defective computer chips. [2]
 - (ii) Find the probability that more than 2 computer chips are defective in this sample. [1]
 - (iii) Find the probability that at most 5 computer chips are defective, given that more than 2 computer chips are defective. [2]
 - (iv) One hundred such samples, each sample containing 20 computer chips, are taken from the firm. Using a suitable approximation, find the probability that there are at most 7 samples which contain more than 2 defective computer chips each. [3]
- 9 (a) In a particular town, 65% of the people watch the show *Jumping Man* and 75% of the people watch the show *Running Woman*. 60% of those who do not watch the show *Jumping Man* watch the show *Running Woman*.
- Find the probability that a person chosen at random from the town
- (i) watches both shows, [2]
 - (ii) does not watch the show *Running Woman* given that the person watches the show *Jumping Man*. [2]
- 9 (b) The weather on any day is classified as either wet or dry. If it is wet on any particular day the probability that it will be wet the next day is 0.6. If it is dry on any particular day the probability that it will be dry the next day is 0.8. Tuesday was a wet day in a particular week.
- (i) Draw a tree diagram to illustrate the possible outcomes from Tuesday to Thursday. [2]
 - (ii) Find the probability that Wednesday is a wet day given that Thursday of that week is wet. [3]

[Turn Over]

- 10 A company sells electric water heaters. A manager claims that their heaters will heat 50 litres of water from a temperature of 10°C to a temperature of 35°C in, on average, no longer than 12 minutes. In order to test this claim, 40 randomly chosen heaters are bought and the time (x minutes) taken by each heater to heat 50 litres of water from 10°C to 35°C is measured and summarised by

$$\sum(x-15) = -106, \quad \sum(x-15)^2 = 318.$$

- (i) Find unbiased estimates of the population mean and variance. [2]
- (ii) Test at the 1% significance level whether the manager's claim is valid. [4]

The company manufactures a new model of water heater and claims that the mean time of the new model of water heater to heat 50 litres of water from a temperature 10°C to a temperature of 35°C is 10 minutes. The time taken by the new model of water heater to heat 50 litres of water is known to have a normal distribution with standard deviation 0.98 minutes. A random sample of 20 water heaters is selected and found to have an average time of k minutes to heat 50 litres of water from 10°C to 35°C . A test at the 5% significance level is carried out on this sample, and the company's claim is accepted. Find the set of possible values of k . [4]

- 11 A supermarket sells two types of curry powder. Type A and Type B curry powders are mixed from two ingredients P and Q . One packet of Type A curry powder contains 2 scoops of ingredient P and 3 scoops of ingredient Q . A packet of Type B curry powder contains 4 scoops of ingredient P and 1 scoop of ingredient Q . The masses, in grams, of ingredients P and Q in scoops have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard deviation
Ingredient P	55	3
Ingredient Q	25	2

- (i) Find the probability that the total mass of two randomly chosen packets of Type A curry powder is within 100 grams of twice the mass of a packet of Type B curry powder. [4]
- (ii) Twenty packets of Type A curry powders are chosen at random. Find the probability that there are greater than 2 but not more than 10 packets of Type A curry powder each weighing more than 0.19 kg. [3]
- (iii) 10 packets of Type A curry powder are packed, in random, into 'long' bags. 5 packets of Type B curry powder are packed, in random, into 'special' bags. Elsa buys a 'long' bag at \$1.20 per 100 gram and Anna buys two 'special' bags at \$1.50 per 100 gram. Find the probability that Anna pays at least \$15 more than Elsa. [4]

- 12 A certain solution discolours when exposed to air. To protect the solution against discolouration, it is treated with a chemical. In an experiment, different quantities, m ml, of the chemical were applied to standard samples of the solution, and the times, t minutes, for the solution to discolour were measured. The results are given in the table.

m	1.3	2.1	2.8	3.5	4.7	5.6	6.5
t	3	13	17	20	22	28	33

- (i) Draw a sketch of the scatter diagram for the data, as shown on your calculator. [1]
- (ii) Find the product moment correlation coefficient and comment on its value in the context of the data. [2]
- (iii) Using a suitable regression line, estimate the quantities of the chemical applied when the time taken for the solution to discolour is 40 minutes. Comment on the reliability of the estimate. Sketch the regression line you have chosen on the scatter diagram. [4]
- (iv) A different type of chemical has been added to delay the process of discolouration of the solution. It is found that the time taken for the solution to discolour has been lengthened by 2 minutes for all the seven samples. Without any further calculations, state any change you would expect in the values of gradient and y – intercept of the regression line in (iii). [1]
- (v) A new pair of values where $m = 5$ and $t = a$ is added to form a new regression line t on m . Find the value of a if the new equation of the regression line t on m is $t = 0.635 + 4.817 m$. Correct your answer to 1 decimal place. [3]

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