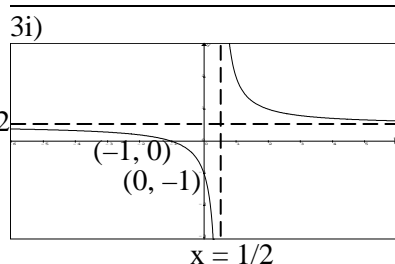


**GCE 'A' Level H2 Maths
Nov 2013 Paper 1**

1i) $x - 2z = 4$
 $2x - 2y + z = 6$
 $5x - 4y + 3z = -9$
 From GC, point of intersection =
 $(-\frac{38}{3}, -\frac{119}{6}, -\frac{25}{3})$.

(ii) $x - 2z = 4$
 $2x - 2y + z = 6$
 $5x - 4y = -9$
 From GC, there is no solution.
 So p, q, r have no common points of intersection.
 Since none of the planes are parallel to each other, the 3 planes form a triangular prism.

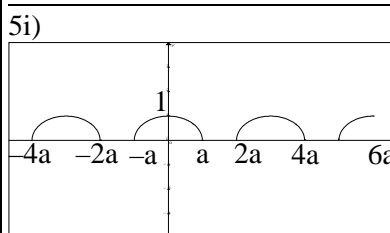
2) $y = \frac{x^2 + x + 1}{x - 1}$
 $xy - y = x^2 + x + 1$
 $x^2 + x(1 - y) + 1 + y = 0$
 For this quadratic equation to have a solution,
 $(1 - y)^2 - 4(1 + y) \geq 0$
 $y^2 - 2y + 1 - 4 - 4y \geq 0$
 $y^2 - 6y - 3 \geq 0$
 $y \leq \frac{6 - \sqrt{48}}{2}$ or $y \geq \frac{6 + \sqrt{48}}{2}$
 $y \leq 3 - 2\sqrt{3}$ or $y \geq 3 + 2\sqrt{3}$



(ii) Let $\frac{x+1}{2x-1} = 1$
 $x + 1 = 2x - 1$
 $x = 2$
 \therefore solution is $x < \frac{1}{2}$ or $x > 2$.

4i) $w^3 = (1 + 2i)^3$
 $= 1 + 3(2i) + 3(2i)^2 + (2i)^3$
 $= 1 + 6i + 3(-4) - 8i$
 $= -11 - 2i$
 (ii) $a(-11 - 2i) + 5(1 + 4i - 4) + 17(1 + 2i) + b = 0$
 $-11a + 2 + b + i(-2a + 54) = 0$

Comparing complex parts:
 $-2a + 54 = 0 \Rightarrow a = 27$
 Comparing real parts:
 $-11a + 2 + b = 0$
 $\Rightarrow b = 295$
 (iii) Since $1 + 2i$ is a root, so $1 - 2i$ is also a root.
 $[z - (1 + 2i)][z - (1 - 2i)]$
 $= z^2 - 2z + 5$
 By comparing coefficients,
 $27z^3 + 5z^2 + 17z + 295$
 $= (z^2 - 2z + 5)(27z + 59)$
 The roots are $1 \pm 2i$ and $-\frac{59}{27}$.



(ii) $\int_{\sqrt{3}a/2}^a f(x) dx$
 $= \int_{\pi/6}^{\pi/3} \sqrt{1 - \sin^2 \theta} a \cos \theta d\theta$
 $= a \int_{\pi/6}^{\pi/3} \cos^2 \theta d\theta$
 $= a \int_{\pi/6}^{\pi/3} \frac{1 + \cos 2\theta}{2} d\theta$
 $= \frac{a}{2} [\theta + \frac{\sin 2\theta}{2}]_{\pi/6}^{\pi/3}$
 $= \frac{a}{2} [\frac{\pi}{3} + \frac{\sqrt{3}}{4} - \frac{\pi}{6} - \frac{\sqrt{3}}{4}]$
 $= \frac{\pi a}{12}$

6i) Equation of plane OAB is $\mathbf{r} = \lambda \mathbf{a} + \mu \mathbf{b}$.
 Since C lies on the plane OAB,
 $\therefore \mathbf{c}$ can be expressed as $\mathbf{c} = \lambda \mathbf{a} + \mu \mathbf{b}$.

(ii) $\vec{ON} = \frac{4\mathbf{a} + 3\mathbf{c}}{7}$
 (iii) Area of ΔONC
 $= \frac{1}{2} \left| \frac{4\mathbf{a} + 3\mathbf{c}}{7} \times \mathbf{c} \right|$
 $= \frac{2}{7} |\mathbf{a} \times \mathbf{c}|$
 $= \frac{2}{7} |\mathbf{a} \times (\lambda \mathbf{a} + \mu \mathbf{b})|$
 $= \frac{2\mu}{7} |\mathbf{a} \times \mathbf{b}|$

Area of ΔOMC
 $= \frac{1}{2} \left| \frac{1}{2} \mathbf{b} \times \mathbf{c} \right|$
 $= \frac{1}{4} |\mathbf{b} \times \mathbf{c}|$
 $= \frac{1}{4} |\mathbf{b} \times (\lambda \mathbf{a} + \mu \mathbf{b})|$
 $= \frac{\lambda}{4} |\mathbf{a} \times \mathbf{b}|$
 $\frac{2\mu}{7} |\mathbf{a} \times \mathbf{b}| = \frac{\lambda}{4} |\mathbf{a} \times \mathbf{b}|$
 $\lambda = \frac{8}{7} \mu$

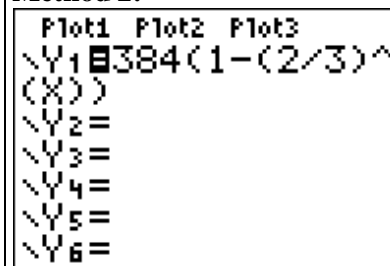
7i) $p = 128 \left(\frac{2}{3}\right)^{n-1}$
 $\ln p = \ln 128 + (n-1) \ln \frac{2}{3}$
 $= 7 \ln 2 + (n-1) \ln 2 - (n-1) \ln 3$
 $= (n+6) \ln 2 + (-n+1) \ln 3$
 $\therefore A = 1, B = 6, C = -1, D = 1$.

(ii) $S_{\infty} = \frac{128}{1 - \frac{2}{3}}$
 $= 384$
 \therefore total length cannot be > 384 cm.

(iii) $S_n = 128 \frac{1 - \left(\frac{2}{3}\right)^n}{1 - \frac{2}{3}}$
 $= 384 \left[1 - \left(\frac{2}{3}\right)^n \right] > 380$
 $\left(\frac{2}{3}\right)^n < \frac{1}{96}$
 $n > \frac{\log \frac{1}{96}}{\log \frac{2}{3}}$
 $= 11.26$

\therefore 12 pieces must be cut off.

Method 2:

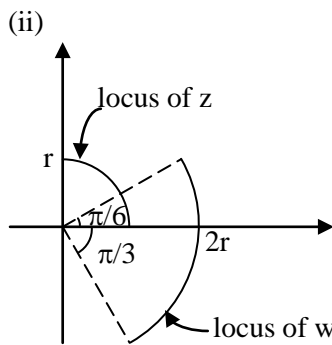


X	Y1
9	374.01
10	377.34
11	379.56
12	381.04
13	382.03
14	382.68
15	383.12

X=12

From GC, 12 pieces must be cut off.

8i) $|w| = |1 - i\sqrt{3}| |z|$
 $= 2r$
 $\arg w = \arg(1 - i\sqrt{3}) + \arg z$
 $= -\frac{\pi}{3} + \theta$



(iii) $10 \arg z - 2 \arg w = \pi$
 $10\theta - 2\left(\theta - \frac{\pi}{3}\right) = \pi$
 $8\theta = \frac{\pi}{3}$
 $\theta = \frac{\pi}{24}$

9) Let $P(n)$ be the statement $\sum_{r=1}^n r(2r^2+1) = \frac{n}{2}(n+1)(n^2+n+1)$.

When $n = 1$:
LHS = $1(2+1) = 3$
RHS = $\frac{1}{2}(2)(3) = 3 = \text{LHS}$

$\therefore P(1)$ is true.
Assume that $P(k)$ is true for some $k \in \mathbb{Z}^+$, i.e.

$$\sum_{r=1}^k r(2r^2+1) = \frac{k}{2}(k+1)(k^2+k+1)$$

To prove that $P(k+1)$ is also true,

i.e. $\sum_{r=1}^{k+1} r(2r^2+1)$

$$= \frac{k+1}{2}(k+2)[(k+1)^2+k+2]$$

$$= \frac{k+1}{2}(k+2)(k^2+3k+3)$$

LHS

$$= \sum_{r=1}^k r(2r^2+1) + (k+1)[2(k+1)^2+1]$$

$$= \frac{k}{2}(k+1)(k^2+k+1) + (k+1)[2(k+1)^2+1]$$

$$= \frac{k+1}{2}[k^3+k^2+k+4(k^2+2k+1)+2]$$

$$= \frac{k+1}{2}(k^3+5k^2+9k+6)$$

$$= \frac{k+1}{2}(k+2)(k^2+3k+3)$$

= RHS

Since $P(1)$ is true, and $P(k)$ is true $\Rightarrow P(k+1)$ is true, by Math Induction, $P(n)$ is true for all $n \in \mathbb{Z}^+$.

(ii) $f(r) - f(r-1)$
 $= 2r^3 + 3r^2 + r + 24 - [2(r-1)^3 + 3(r-1)^2 + r-1 + 24]$
 $= 2r^3 + 3r^2 + r + 24 - [2(r^3 - 3r^2 + 3r - 1) + 3r^2 - 6r + 3 + r - 1 + 24]$
 $= 2r^3 + 3r^2 + r + 24 - [2r^3 - 3r^2 + 3r - 1 + 3r^2 - 6r + 3 + r - 1 + 24]$
 $= 2r^3 + 3r^2 + r + 24 - [2r^3 - 3r^2 + 3r - 1 + 3r^2 - 6r + 3 + r - 1 + 24]$
 $= 6r^2$

$$6 \sum_{r=1}^n r^2 = \sum_{r=1}^n [f(r) - f(r-1)]$$

$$= f(1) - f(0) + f(2) - f(1)$$

\vdots

$$+ f(n) - f(n-1)$$

$$= f(n) - f(0)$$

$$= 2n^3 + 3n^2 + n + 24 - 24$$

$$\sum_{r=1}^n r^2 = \frac{1}{6}(2n^3 + 3n^2 + n)$$

$$= \frac{n}{6}(2n^2 + 3n + 1)$$

$$= \frac{n}{6}(2n+1)(n+1)$$

(iii) $\sum_{r=1}^n f(r)$

$$= \sum_{r=1}^n (2n^3 + 3n^2 + n + 24)$$

$$= \sum_{r=1}^n r(2r^2+1) + 3 \sum_{r=1}^n r^2 + \sum_{r=1}^n 24$$

$$= \frac{n}{2}(n+1)(n^2+n+1) + \frac{n}{2}(2n+1)(n+1) + 24n$$

10i) $\int \frac{1}{3-2z} dz = \int dx$

$$-\frac{1}{2} \ln(3-2z) = x + C$$

$$\ln(3-2z) = -2x - 2C$$

$$3-2z = e^{-2x-2C}$$

$$2z = 3 - e^{-2x-2C}$$

$$z = \frac{3}{2} - \frac{1}{2} e^{-2x} e^{-2C}$$

$$= \frac{3}{2} + Ae^{-2x} \text{ where } A = \frac{1}{2} e^{-2C}$$

(ii) $\frac{dy}{dx} = \frac{3}{2} + Ae^{-2x}$

$$y = \frac{3}{2}x - \frac{A}{2}e^{-2x} + D$$

(iii) $\frac{d^2y}{dx^2} = -2Ae^{-2x}$

$$= -2\left(\frac{dy}{dx} - \frac{3}{2}\right)$$

$$= -2\frac{dy}{dx} + 3$$

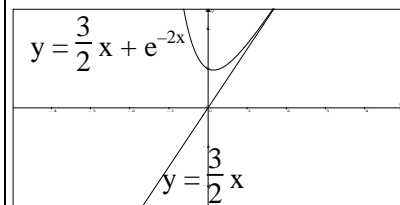
$$\therefore a = -2, b = 3$$

(iv) Let $A = 0$.

2 members of the family are $y = \frac{3}{2}x$ and $y = \frac{3}{2}x + 1$.

$$y = \frac{3}{2}x \text{ and } y = \frac{3}{2}x + 1$$

Let $A = -2, D = 0$: $y = \frac{3}{2}x + e^{-2x}$.



11i) $\frac{dx}{dt} = 6t, \frac{dy}{dt} = 6t^2$

$$\frac{dy}{dx} = \frac{6t^2}{6t} = t$$

Equation of tangent is

$$y - 2t^3 = t(x - 3t^2)$$

$$y = tx - 3t^3 + 2t^3$$

$$= tx - t^3$$

(ii) Equate $y = px - p^3$ with

$$y = qx - q^3$$

$$px - p^3 = qx - q^3$$

$$\begin{aligned}
 (p-q)x &= p^3 - q^3 \\
 x &= \frac{(p-q)(p^2 + pq + q^2)}{p-q} \\
 &= p^2 + pq + q^2 \\
 y &= p(p^2 + pq + q^2) - p^3 \\
 &= p(p^2 + pq) \\
 &= pq(p+q) \\
 y^2 + 1 &= p^2 q^2 (p+q)^2 + 1 \\
 &= (p+q)^2 + 1 \\
 &= p^2 + 2pq + q^2 + 1 \\
 &= p^2 + 2pq + q^2 - pq \\
 &= p^2 + q^2 + pq \\
 &= x
 \end{aligned}$$

∴ R lies on the curve $x = y^2 + 1$.

(iii) Substitute $x = 3t^2$, $y =$

$2t^3$ into $x = y^2 + 1$:

$$\begin{aligned}
 3t^2 &= (2t^3)^2 + 1 \\
 &= 4t^6 + 1
 \end{aligned}$$

$$\therefore 4t^6 - 3t^2 + 1 = 0$$

$$(t^2 + 1)(4t^4 - 4t^2 + 1) = 0$$

$$(t^2 + 1)(2t^2 - 1)^2 = 0$$

$$t^2 = \frac{1}{2}$$

$$t = \frac{1}{\sqrt{2}} \text{ since } y > 0$$

$$x = \frac{3}{2}, y = \frac{1}{\sqrt{2}}$$

$$\therefore M = \left(\frac{3}{2}, \frac{1}{\sqrt{2}} \right)$$

(iv) Area

$$= \int_0^{3/2} y \, dx - \int_1^{3/2} \sqrt{x-1} \, dx$$

$$= \int_0^{1/\sqrt{2}} 2t^3 \cdot 6t \, dt - \left[\frac{(x-1)^{3/2}}{3/2} \right]_1^{3/2}$$

$$\int_1^{3/2}$$

$$= 12 \int_0^{1/\sqrt{2}} t^4 \, dt - \frac{2}{3} \left[\frac{1}{2\sqrt{2}} \right]$$

$$= 12 \left[\frac{t^5}{5} \right]_0^{1/\sqrt{2}} - \frac{1}{3\sqrt{2}}$$

$$= \frac{3}{5\sqrt{2}} - \frac{1}{3\sqrt{2}}$$

$$= \frac{4}{15\sqrt{2}}$$

**GCE 'A' Level H2 Maths
Nov 2013 Paper 2**

1(i) $R_g = \mathbb{R} \not\subseteq \mathbb{R} \setminus \{1\} = D_f$

∴ fg does not exist.

(ii) gf(x)

$$= g\left(\frac{2+x}{1-x}\right)$$

$$= 1 - 2\left(\frac{2+x}{1-x}\right)$$

$$= 1 - \frac{4+2x}{1-x}$$

$$= \frac{1-x-(4+2x)}{1-x}$$

$$= -\frac{3+3x}{1-x}$$

Let $y = (gf)^{-1}(5)$

$$gf(y) = 5$$

$$-\frac{3+3y}{1-y} = 5$$

$$-3 - 3y = 5 - 5y$$

$$2y = 8$$

$$y = 4$$

(2i) Length of side of base

$$= a - 2 \frac{x}{\tan 30^\circ}$$

$$= a - \frac{2x}{1/\sqrt{3}}$$

$$= a - 2x\sqrt{3}$$

Volume of prism, V

$$= \frac{1}{2} (a - 2x\sqrt{3})^2 \sin 60^\circ x$$

$$= \frac{x\sqrt{3}}{4} (a - 2x\sqrt{3})^2$$

$$\text{(ii) } \frac{dV}{dx} = \frac{\sqrt{3}}{4} (a - 2x\sqrt{3})^2$$

$$+ \frac{x\sqrt{3}}{4} 2(a - 2x\sqrt{3})(-2\sqrt{3})$$

$$= \frac{\sqrt{3}}{4} (a - 2x\sqrt{3})(a - 2x\sqrt{3} -$$

$$4x\sqrt{3})$$

$$= \frac{\sqrt{3}}{4} (a - 2x\sqrt{3})(a - 6x\sqrt{3}) = 0$$

$$x = \frac{a}{2\sqrt{3}} \text{ or } \frac{a}{6\sqrt{3}}$$

$$\frac{d^2V}{dx^2} = \frac{\sqrt{3}}{4} (-2\sqrt{3})(a - 6x\sqrt{3})$$

$$+ \frac{\sqrt{3}}{4} (a - 2x\sqrt{3})(-6\sqrt{3})$$

$$= -\frac{3}{2} (a - 6x\sqrt{3} + 3a - 6x\sqrt{3})$$

$$= -\frac{3}{2} (4a - 12x\sqrt{3})$$

$$= 6(3x\sqrt{3} - a)$$

$$\text{When } x = \frac{a}{2\sqrt{3}}, \frac{d^2V}{dx^2} = 3a > 0.$$

$$\text{When } x = \frac{a}{6\sqrt{3}}, \frac{d^2V}{dx^2} = -3a < 0.$$

∴ maximum V

$$= \frac{a}{6\sqrt{3}} \frac{\sqrt{3}}{4} \left(a - \frac{a}{3}\right)^2$$

$$= \frac{a}{24} \left(\frac{2a}{3}\right)^2$$

$$= \frac{a^2}{54}$$

$$\text{(3i) } f'(x) = \frac{2 \cos x}{1 + 2 \sin x}$$

$f''(x) =$

$$\frac{(1+2 \sin x)(-2 \sin x) - (2 \cos x)^2}{(1+2 \sin x)^2}$$

$$= \frac{-2 \sin x - 4 \sin^2 x - 4 \cos^2 x}{(1+2 \sin x)^2}$$

$$= \frac{-2 \sin x - 4}{(1+2 \sin x)^2}$$

$f'''(x) =$

$$= \frac{(1+2 \sin x)^2(-2 \cos x) + (2 \sin x + 4)2(1+2 \sin x)2 \cos x}{(1+2 \sin x)^4}$$

$$f(0) = 0$$

$$f'(0) = 2$$

$$f''(0) = -4$$

$$f'''(0) = -2 + 16 = 14$$

$$f(x) = 0 + 2x - \frac{4}{2}x^2 + \frac{14}{6}x^3 + \dots$$

$$= 2x - 2x^2 + \frac{7}{3}x^3 + \dots$$

(ii) $e^{ax} \sin nx$

$$= (1 + ax + \frac{a^2x^2}{2} + \dots)(nx - \frac{n^3x^3}{6}$$

+...)

$$= nx + anx^2 + \dots = 2x - 2x^2 + \dots$$

$$\therefore n = 2, a = -1$$

Third non-zero term

$$= \frac{a^2n}{2}x^3 - \frac{n^3x^3}{6}$$

$$= x^3 - \frac{4}{3}x^3$$

$$= -\frac{1}{3}x^3$$

$$(4i) \quad \cos \theta = \frac{\left| \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 3 \\ 2 \end{pmatrix} \right|}{\sqrt{9} \sqrt{49}} = \frac{|-12 - 6 + 2|}{3 \times 7} = \frac{16}{21}$$

$$\theta = 40.4^\circ$$

$$(ii) \quad \begin{aligned} 2x - 2y + z &= 1 \\ -6x + 3y + 2z &= -1 \end{aligned}$$

```

SYSTEM MATRIX (2x4)
[ 2  -2  1  |  1 ]
[ -6  3  2  | -1 ]

(1,1)=2
(MAIN)MODE(SCLR)LOAD(SOLVE)
SOLUTION SET
x1 = -1/6 + 7/6 x3
x2 = -2/3 + 5/3 x3
x3 = x3

(MAIN)MODE(SYSM)STO(VARF)

```

From GC, equation of line l is r

$$= \begin{pmatrix} -1/6 \\ -2/3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 7/6 \\ 5/3 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$$

(iii) Distance from A to p_1

$$= \frac{\left| \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \\ c \end{pmatrix} - 1 \right|}{\sqrt{9}} = \frac{|1+c|}{3}$$

Distance from A to p_2

$$= \frac{\left| \begin{pmatrix} -6 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \\ c \end{pmatrix} + 1 \right|}{\sqrt{49}} = \frac{|2c-14|}{7}$$

$$\frac{|1+c|}{3} = \frac{|2c-14|}{7}$$

$$49(1+c)^2 = 9(2c-14)^2$$

$$49(1+2c+c^2) = 9(4c^2 - 56c + 196)$$

$$13c^2 + 602c - 1715 = 0$$

$$c = -49, 2.69$$

(5i) Print the names of all

100 000 employees on slips of paper and put in a box. Randomly pick 90 names from the box.

There may be too many representatives from some countries, and none from some countries.

(ii) Use stratified sampling. The number of employees from each country to be invited to the party is proportional to the number of employees in that country. Eg, if Singapore has n employees, then invite $\frac{n}{100\,000} \times 90$ employees from Singapore. Use random sampling to pick required number of employees from each country.

$$(6) \quad P(Y < 2a) = 0.95$$

$$P\left(Z < \frac{2a - \mu}{\sigma}\right) = 0.95$$

$$\frac{2a - \mu}{\sigma} = 1.64485$$

$$2a - \mu = 1.64485\sigma \quad \text{---(1)}$$

$$P(Y < a) = 0.25$$

$$P\left(Z < \frac{a - \mu}{\sigma}\right) = 0.25$$

$$\frac{a - \mu}{\sigma} = -0.67449$$

$$a - \mu = -0.67449\sigma \quad \text{---(2)}$$

$$(1) - (2) \Rightarrow a = 2.31934\sigma$$

$$\mu = a + 0.67449 \frac{a}{2.31934} = 1.29a$$

(7i) The probability of picking a packet containing a free gift is constant.

Whether a packet contains a free gift or not is independent of other packets.

$$(ii) \quad F \sim B\left(20, \frac{1}{20}\right)$$

$$P(F = 1) = 0.377$$

$$(iii) \quad F \sim B\left(60, \frac{1}{20}\right)$$

Since $n = 60 > 50$ and $np = 3 < 5$, $F \sim \text{Po}(3)$ approximately.

$$P(F \geq 5) = 1 - P(F \leq 4) = 0.185$$

$$(8i) \quad P(B | A') = \frac{P(B \cap A')}{P(A')}$$

$$\therefore P(B \cap A') = 0.8 \times 0.3 = 0.24$$

(ii)

$$P(A') = P(A' \cap B) + P(A' \cap B')$$

$$\therefore P(A' \cap B') = 0.3 - 0.24 = 0.06$$

$$(iii) \quad P(A | B') = \frac{P(A \cap B')}{P(B')}$$

$$0.88 = \frac{P(A \cap B')}{P(A \cap B') + P(A' \cap B')}$$

$$0.88 P(A \cap B') + 0.88 \times 0.06 = P(A \cap B')$$

$$0.12 P(A \cap B') = 0.0528$$

$$P(A \cap B') = 0.44$$

$$P(A) = P(A \cap B) + P(A \cap B')$$

$$0.7 = P(A \cap B) + 0.44$$

$$P(A \cap B) = 0.26$$

(9i)

L1	L2	L3	1
FL	-----	-----	
12.5			
11			
11			
12.5			
12.6			
15.6			
L1(1) = 14			
1-Var Stats			
$\bar{x} = 12.8$			
$\Sigma x = 102.4$			
$\Sigma x^2 = 1326.86$			
$Sx = 1.518457864$			
$\sigma x = 1.420387271$			
$\downarrow n = 8$			

From GC, unbiased estimate of

$\mu = 12.8$ and unbiased estimate

$$\text{of } \sigma^2 = 1.51846^2$$

$$= 2.3057$$

$$= 2.31$$

(ii) We assume that 2.31 is a good estimate of the unknown the population variance.

$$H_0: \mu = 13.8$$

$$H_1: \mu < 13.8$$

```

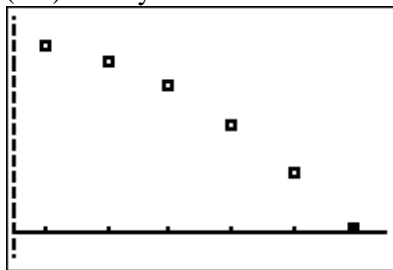
T-Test
Inpt: DATA Stats
μ₀: 13.8
List: L1
Freq: 1
μ: ≠ μ₀ < μ₀ > μ₀
Calculate Draw

μ < 13.8
t = -1.862697142
P = .0523978347
x̄ = 12.8
Sx = 1.518457864
n = 8

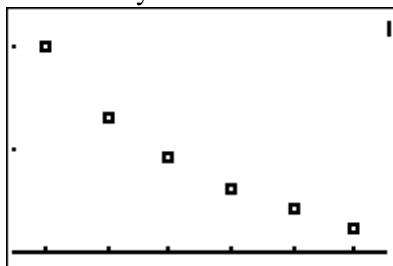
```

Since p-value = 0.0524 > 0.05, we do not reject H_0 . There is insufficient evidence at 5% significance level to say that the distance travelled per litre of fuel is too high.

(10i) A: $y = a + bx^2$



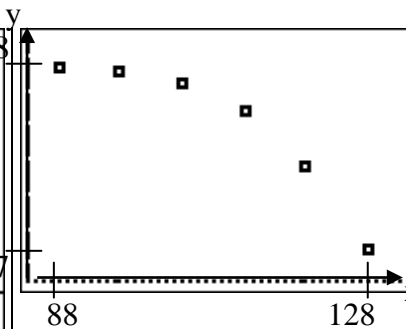
B: $y = c + d \ln x$



C: $y = e + \frac{f}{x}$



(ii)



(iii) Since the data points seem to lie close to a curve with negative gradient and concave downwards, model A is the most appropriate.

L1	L2	Σ	Σ 3
88	148	7744	
96	147	9216	
104	144	10816	
112	138	12544	
120	126	14400	
128	107	16384	

L3 = "L1 2"

```

LinReg(ax+bx)
Xlist: L3
Ylist: L2
FreqList:
Store RegEQ:
Calculate

```

```

LinReg
y = ax + b
a = -.0046197847
b = 189.7475283
r² = .882101978
r = -.9392028418

```

From GC, $r = -0.939$

(iv)

From GC, regression line is

$$y = 189.748 - 0.0046198x^2$$

Distance travelled

$$= 189.748 - 0.0046198(110^2)$$

$$= 133.8$$

$$= 134 \text{ km}$$

$$(11i) \frac{26 \times 25 \times 24 \times 9 \times 8}{26 \times 26 \times 26 \times 9 \times 9} = 0.789$$

$$(ii) \frac{1 - P(\text{same digits})}{2}$$

$$= \frac{1 - \frac{9}{9^2}}{2}$$

$$= \frac{4}{9}$$

(iii) Case 1: exactly 2 same letters and 2 different digits

$$\text{No. of ways} = 26 \times 25 \times \frac{3!}{2!} \times {}^9P_2 = 140\,400$$

Case 2: all letters different and 2 same digits

$$\text{No. of ways} = {}^{26}P_3 \times 9 = 140\,400$$

Case 3: 3 same letters and 2 same digits

$$\text{No. of ways} = 26 \times 9 = 234$$

Probability

$$= \frac{140\,400 + 140\,400 + 234}{26 \times 26 \times 26 \times 9 \times 9} = 0.197$$

(iv) Case 1: 2 different consonants, 1 vowel and 1 even digit

$$\text{No. of ways} = {}^{21}C_2 {}^5C_1 3! {}^4C_1 {}^5C_1 2! = 252\,000$$

Case 2: 2 same consonants, 1 vowel and 1 even digit

$$\text{No. of ways} = {}^{21}C_1 {}^5C_1 \frac{3!}{2!} {}^4C_1 {}^5C_1 2! = 12\,600$$

$$\text{Probability} = \frac{252\,000 + 12\,600}{26 \times 26 \times 26 \times 9 \times 9} = 0.186$$

(12i) The mean no. of absentees per unit time is constant.

The probability of an employee being absent is independent of other employees.

The mean no. of absentees may be higher during an epidemic.

If an employee is ill, his illness may be contagious and will increase the probability of other employees becoming ill.

(ii) Let A = no. of absentees from Admin Dept on n days.

$A \sim \text{Po}(1.2n)$

$$P(A = 0) = e^{-1.2n} < 0.01$$

$$-1.2n < \ln 0.01$$

$$n > 3.8$$

Smallest no. of days = 4

(iii) Let T = no. of days of absence from the 2 depts in 5 days

$$T \sim \text{Po}((1.2 + 2.7)5) \\ = \text{Po}(19.5)$$

$$P(T > 20) = 1 - P(T \leq 20) \\ = 0.397$$

(iv) Let S = no. of days of absence from the 2 depts in 60 days.

$$S \sim \text{Po}((1.2 + 2.7)60) \\ = \text{Po}(234)$$

Since $\lambda = 234 > 10$, $S \sim N(234, 234)$ approximately.

$$P(200 \leq S \leq 250) \\ = P(199.5 \leq S \leq 250.5) \\ = 0.848$$
