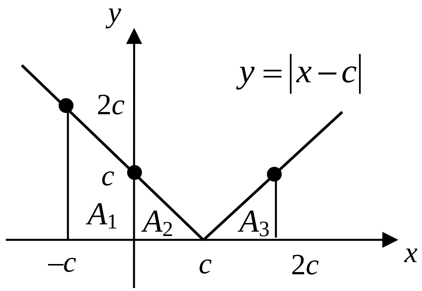
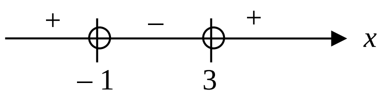
1	System of Linear Equations			
	Assessment Objectives	**Solution**		**Feedback**
	• To use a system of linear equations to model and solve practical problems • To find the numerical solution of a system of linear equations using a graphic calculator. • To perform basic differentiation and make use of the fact that gradient of a curve is 0 at turning points.	$y=ax+b+\dfrac{c}{x+3}$ --------- (*) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=a-\dfrac{c}{(x+3)^2}$ Substituting the x- and y-coordinates into eqn (*), $\left(1,\ \dfrac{5}{16}\right)$: $a+b+\dfrac{1}{4}c=\dfrac{5}{16}$ ------ (1) $\left(2,\ \dfrac{1}{4}\right)$: $2a+b+\dfrac{1}{5}c=\dfrac{1}{4}$ ------ (2) Since there is a turning point at $\left(2,\ \dfrac{1}{4}\right)$, $x=2$ would be a solution to $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$. $\Rightarrow a-\dfrac{1}{25}c=0$ ------ (3) Solving with G.C., $a=\dfrac{1}{4}$ (or 0.25) $b=-\dfrac{3}{2}$ (or – 1.5) $c=\dfrac{25}{4}$ (or 6.25)	**[M1]** **[M½]** **[M½]** **[M1]** **[A1]**	Subtract ½ for every error.

2	Techniques of Integration			
	Assessment Objectives	**Solution**		**Feedback**
	• Able to interpret $\int_a^b \mathrm{f}(x)\mathrm{d}x$ as the area under the curve $y = \mathrm{f}(x)$ between $x = a$ and $x = b$ **[HOT]**	**Note:** $\lvert x-c\rvert = \begin{cases} x-c, & \text{if } x \geq c \\ c-x, & \text{if } x < c \end{cases}$		
		$\int_{-c}^{0} \lvert x-c\rvert \mathrm{d}x = \int_{-c}^{0} c-x\mathrm{d}x$	**[M½]**	
		$= \left[cx - \frac{x^2}{2} \right]_{-c}^{0}$	**[M½]**	
		$= -\left[c(-c) - \frac{(-c)^2}{2} \right]$		
		$= c^2 + \frac{1}{2}c^2$		
		$= \frac{3}{2}c^2$	**[A½]**	
		$\int_{0}^{2c} \lvert x-c\rvert \mathrm{d}x = \int_{0}^{c} c-x\mathrm{d}x + \int_{c}^{2c} x-c\mathrm{d}x$	**[M½]**	
		$= \left[cx - \frac{x^2}{2} \right]_{0}^{c} + \left[\frac{x^2}{2} - cx \right]_{c}^{2c}$	**[M½]**	
		$= c^2 - \frac{c^2}{2} + \left(\frac{4c^2}{2} - 2c^2 \right) - \left(\frac{c^2}{2} - c^2 \right)$		
		$= c^2$	**[A½]**	
		$\int_{-c}^{0} \lvert x-c\rvert \mathrm{d}x = k\int_{0}^{2c} \lvert x-c\rvert \mathrm{d}x \Leftrightarrow \frac{3}{2}c^2 = kc^2$		
		$\therefore k = \frac{3}{2}$	**[A1]**	

		Alternative: 	[B1]	
		$\int_{-c}^{0} \|x-c\| \mathrm{d}x = k\int_{0}^{2c} \|x-c\| \mathrm{d}x$ Area A_1 = k (Area A_2 + Area A_3) $\frac{1}{2}c(2c+c) = k\left(\frac{1}{2}c(c)+\frac{1}{2}c(c)\right)$	[M2]	
		$\frac{1}{2}\left(3c^2\right) = kc^2$ $k = \frac{3}{2}$	[A1]	

3 Mathematical Induction

	Assessment Objectives	Solution		Feedback
		Let P_n be the statement $\sum_{r=1}^{n} r(3r-2)=\frac{n}{2}(n+1)(2n-1)$ for $n \in \mathbb{Z}^+$	**[B½]**	
		When n = 1: L.H.S. = 1(3 – 2) = 1		
		R.H.S. = $\frac{1}{2}$ (2)(1) = 1		
		$\therefore$ P_1 is true and this forms the basis for induction.	**[M1]**	
		Assume P_k is true for some $k \in \mathbb{Z}^+$,		
		i.e. $\sum_{r=1}^{k} r(3r-2)=\frac{k}{2}(k+1)(2k-1)$	**[M1]**	
		R.T.P P_{k+1} is true, i.e. $\sum_{r=1}^{k+1} r(3r-2)=\frac{(k+1)}{2}(k+2)(2k+1)$		
		L.H.S. $= \sum_{r=1}^{k+1} r(3r-2)$		
		$= \sum_{r=1}^{k} r(3r-2)+(k+1)\left[3(k+1)-2\right]$	**[M1]**	
		$=\frac{k}{2}(k+1)(2k-1)+(k+1)(3k+1)$		
		$= \frac{(k+1)}{2}[k(2k-1)+2(3k+1)]$		
		$=\frac{(k+1)}{2}(2k^2-k+6k+2)$		
		$=\frac{(k+1)}{2}(2k^2+5k+2)$	**[A1]**	
		$=\frac{(k+1)}{2}(k+2)(2k+1)$ = R.H.S.		
		P_k true $\Rightarrow P_{k+1}$ is also true		
		Since P_1 is true, hence by MI, P_n is true for $n \in \mathbb{Z}^+$.	**[B½]**	

4	Inequalities			
	Assessment Objectives	**Solution**		**Feedback**
	• To manipulate and solve inequalities of the form $\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)} > 0$ where f(x) and g(x) are quadratic expressions that are either factorisable or always positive.	$\dfrac{x+4}{3+2x-x^2} < 1$ ----------- (*)		
		$\dfrac{x+4}{3+2x-x^2} - 1 < 0$	**[M1]**	
		$\dfrac{x+4}{3+2x-x^2} - \dfrac{3+2x-x^2}{3+2x-x^2} < 0$		
		$\dfrac{x^2-x+1}{3+2x-x^2} < 0$	**[M1]**	
		$\dfrac{x^2-x+1}{x^2-2x-3} > 0$		
		Since $x^2-x+1 = \left(x-\dfrac{1}{2}\right)^2 + \dfrac{3}{4} > 0$ for any real x (**OR** coefficient of $x^2 = 1 > 0$ and discriminant, $b^2-4ac = (-1)^2 - 4(1)(1) = -3 < 0$ imply that $x^2-x+1 > 0$ for any real x) **Award 1 mark as long as student indicates 'numerator always positive'.**	**[M1]**	
		$\therefore \dfrac{1}{x^2-2x-3} > 0$	**[M1]**	
		$\dfrac{1}{(x+1)(x-3)} > 0$		
			**[A1]**	
		$\therefore x < -1$ or $x > 3$		

	• To deduce the solution of an inequality by making suitable substitution from an inequality solved previously. **[HOT]**	By replacing x with $\left(-x^2\right)$ in (*), we have: $\frac{-x^2+4}{3-2x^2-x^4}<1$ $\frac{x^2-4}{x^4+2x^2-3}<1$ which is what we need to solve. From earlier part, $-x^2<-1$ or $-x^2>3$ (N.A., since $-x^2\le 0$ for all real x) $x^2>1$ $x<-1$ or $x>1$	**[M1]** **[A1]**	

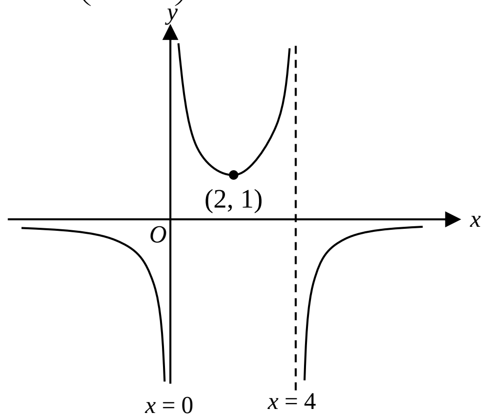
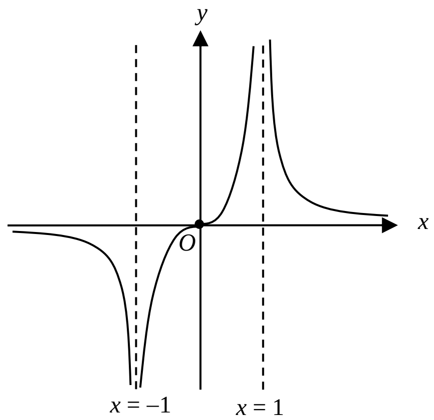
5 Recurrence Relations

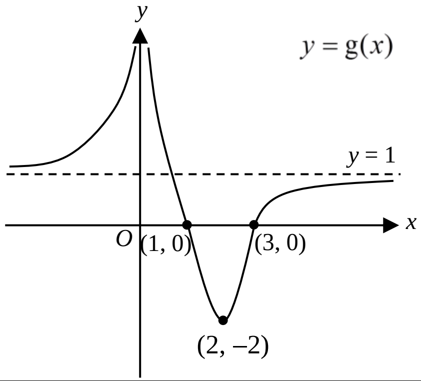
Assessment Objectives	Solution		Feedback
• To use G.C. to find the root	**(i)** $\alpha = 1.873$	**[B1]**	
• To recognise that when sequence converges, $x_{n+1} = x_n$ (Ability to use the given sketch)	**(ii)** For a sequence to converge, as $n \to \infty$, $x_n \to l$ and $x_{n+1} \to l$. $x_{n+1} = \ln\left[3 + \left(x_n\right)^2\right]$ $l = \ln(3 + l^2)$ $e^l = 3 + l^2$ $3 - e^l + l^2 = 0$ $\Rightarrow l = \alpha$ (proved)	**[B1]** **[B1]**	
[HOT]	**(iii)** Consider $e^{x_{n+1}} - e^{x_n} = e^{\ln\left[3+\left(x_n\right)^2\right]} - e^{x_n}$ $= 3 + \left(x_n\right)^2 - e^{x_n}$ If $x_n < \alpha$, $y > 0$ $3 - e^{x_n} + {x_n}^2 > 0$ $e^{x_{n+1}} > e^{x_n}$ $x_{n+1} > x_n$ (proved) If $x_n > \alpha$, $y^{`} < 0$ $3 - e^{x_n} + {x_n}^2 < 0$ $e^{x_{n+1}} < e^{x_n}$ $x_{n+1} < x_n$ (proved)	**[B1]** **[B½]** **[B½]** **[B½]** **[B½]**	

	[HOT]	**(iv)**	When $x_1 = 0$, the sequence increases and converges to 1.873 (i.e. α). When $x_1 = 3$, the sequence decreases and converges to 1.873 (i.e. α). $x_1 = 0$ satisfies $x_1 < \alpha$, thus $x_{n+1} > x_n$ i.e. the sequence increases and converges to α. $x_1 = 3$ satisfies $x_1 > \alpha$, thus $x_{n+1} < x_n$ i.e. the sequence decreases and converges to α.	**[B1]** **[B1]** **[B½]** **[B½]**	

6	Sigma Notation/ Method of Difference			
	Assessment Objectives	**Solution**		**Feedback**
		$\dfrac{1}{r(r+1)(r+2)}=\dfrac{A}{r}+\dfrac{B}{(r+1)}+\dfrac{C}{(r+2)}$ $1=A(r+1)(r+2)+Br(r+2)+Cr(r+1)$ Substituting $r=0$, $A=\dfrac{1}{2}$ Substituting $r=-1$, $B=-1$ Substituting $r=-2$, $C=\dfrac{1}{2}$	**[M½]** **[A1½]**	
	• To express in partial fractions & to use recurrence to solve the problem	**(a)** $\displaystyle\sum_{r=1}^{n}\frac{1}{r(r+1)(r+2)}=\frac{1}{2}(1)-\left(\frac{1}{2}\right)+\frac{1}{2}\left(\frac{1}{3}\right)$ $+\dfrac{1}{2}\left(\dfrac{1}{2}\right)-\left(\dfrac{1}{3}\right)+\dfrac{1}{2}\left(\dfrac{1}{4}\right)$ $+\dfrac{1}{2}\left(\dfrac{1}{3}\right)-\left(\dfrac{1}{4}\right)+\dfrac{1}{2}\left(\dfrac{1}{5}\right)$ $+\ ....$ $+\dfrac{1}{2}\left(\dfrac{1}{n-2}\right)-\left(\dfrac{1}{n-1}\right)+\dfrac{1}{2}\left(\dfrac{1}{n}\right)$ $+\dfrac{1}{2}\left(\dfrac{1}{n-1}\right)-\dfrac{1}{n}+\dfrac{1}{2}\left(\dfrac{1}{n+1}\right)$ $+\dfrac{1}{2}\left(\dfrac{1}{n}\right)-\left(\dfrac{1}{n+1}\right)+\dfrac{1}{2}\left(\dfrac{1}{n+2}\right)$ $=\dfrac{1}{4}-\dfrac{1}{2}\left(\dfrac{1}{n+1}\right)+\dfrac{1}{2}\left(\dfrac{1}{n+2}\right)$	**[M1]** **[A1]**	

	• To apply Sigma notation to the problem and to find Sum to infinity	**(b)(i)**	$\sum_{r=2}^{\infty}\frac{1}{r(r+1)(r+2)}$ $=\sum_{r=1}^{\infty}\frac{1}{r(r+1)(r+2)}-\frac{1}{1(1+1)(1+2)}$ $=\frac{1}{4}-\frac{1}{6}$ $=\frac{1}{12}$	**[M1]** **[A1]**	
	• To link to previous part by a suitable substitution. **[HOT]**	**(ii)**	$\sum_{r=3}^{\infty}\frac{1}{r(r^2-1)}=\sum_{r=3}^{\infty}\frac{1}{(r-1)r(r+1)}$ Let $r = s + 1$ $\sum_{s+1=3}^{\infty}\frac{1}{(s)(s+1)(s+2)}=\sum_{s=2}^{\infty}\frac{1}{(s)(s+1)(s+2)}$ $=\frac{1}{12}$	**[M1]** **[M1]** **[A1]**	

7	Transformations		
	Assessment Objectives	**Solution**	**Feedback**
	• To interpret that $y = \mathrm{f}(ax + b)$ as a composition of two transformations of graphs.	**(a)(i)** $y = \mathrm{f}\left(\frac{1}{2}x - 1\right)$  Shape **[B1]** Asymptotes **[B½]** Turning point **[B½]**	
	• To sketch $y = \mathrm{f}'(x)$ given the graph $y = \mathrm{f}(x)$	**(ii)** $y = \mathrm{f}'(x)$  Shape [**B1]** Asymptotes **[B½]** Intercept **[B½]**	

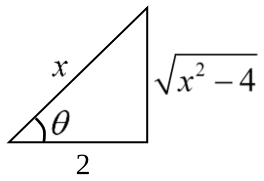
	• To obtain the graph of $y = \mathrm{g}(x)$ given $y = \lvert \mathrm{g}(x) \rvert$ and $y^2 = \mathrm{f}(x)$. **[HOT]**	**(b)**  Shape **[B1]** Asymptotes **[B1]** Intercepts **[B1]**		

8	Maclaurin series			
	Assessment Objectives	**Solution**		**Feedback**
	• To find the Maclaurin series	**(i)** $y=\sin^{-1}x$ $\sin(y)=x$ Differentiate with respect to x: $\left[\cos(y)\right]\dfrac{\mathrm{d}y}{\mathrm{d}x}=1$ Differentiate with respect to x: $\left[\cos(y)\right]\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\dfrac{\mathrm{d}y}{\mathrm{d}x}\left[-\sin(y)\dfrac{\mathrm{d}y}{\mathrm{d}x}\right]=0$ $\left[\cos(y)\right]\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-\left[\sin(y)\right]\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2=0$ Differentiate with respect to x: $\left[\cos(y)\right]\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\left[-\sin(y)\dfrac{\mathrm{d}y}{\mathrm{d}x}\right]+$ $\left[-\sin(y)\right]\left[2\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right]+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2\left(-\cos(y)\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=0$ $\left[\cos(y)\right]\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}-3\left[\sin(y)\right]\dfrac{\mathrm{d}y}{\mathrm{d}x}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-\left[\cos(y)\right]\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^3=0$ When $x=0$ $y=\sin^{-1}0=0$ $(\cos 0)\dfrac{\mathrm{d}y}{\mathrm{d}x}=1\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=1$ $(\cos 0)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-(\sin 0)(1)^2=0\Rightarrow\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ $(\cos 0)\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}-3(\sin 0)(1)(0)-(\cos 0)(1)^3=0\Rightarrow\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=1$	**[M1]** **[M1]** **[M1]** **[A1]**	

		Hence $y=0+1(x)+0\left(\frac{x^2}{2!}\right)+1\left(\frac{x^3}{3!}\right)+\cdots$ $y=x+\frac{x^3}{6}+\cdots$	[A1]	
	• To find the binomial expansion of a series	**(ii)** $\frac{x}{\left(a+bx^2\right)}=x\left(a+bx^2\right)^{-1}$ $=\frac{1}{a}x\left(1+\frac{b}{a}x^2\right)^{-1}$ $=\frac{1}{a}x\left(1-\frac{b}{a}x^2+\cdots\right)$ $=\frac{1}{a}x-\frac{b}{a^2}x^3+\cdots$ Comparing $a=1, b=-\frac{1}{6}$	**[M1]** **[M1]** **[M1]** **[A1]**	•

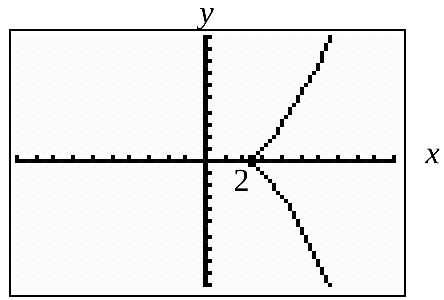
9	Vectors			
	Assessment Objectives	**Solution**		**Feedback**
	• To perform addition and subtraction of vectors, multiplication of a vector by a scalar and understand their geometrical interpretations. • To understand concepts of scalar product of vectors. • To interpret the scalar product and understand its relationship with the angle between the two direction vectors. **[HOT]**	**(a)** $\dfrac{\mathbf{r}\cdot\mathbf{a}}{\mathbf{r}\cdot\mathbf{b}}=\dfrac{\mu\lvert\mathbf{a}\rvert^2+\lambda\mathbf{a}\cdot\mathbf{b}}{\lambda\lvert\mathbf{b}\rvert^2+\mu\mathbf{a}\cdot\mathbf{b}}$ Given $\lambda:\mu=\lvert\mathbf{a}\rvert:\lvert\mathbf{b}\rvert\Rightarrow\dfrac{\lambda}{\mu}=\dfrac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert}$ $\dfrac{\mathbf{r}\cdot\mathbf{a}}{\mathbf{r}\cdot\mathbf{b}}=\dfrac{\mu\left(\lvert\mathbf{a}\rvert^2+\dfrac{\lambda}{\mu}\mathbf{a}\cdot\mathbf{b}\right)}{\mu\left(\dfrac{\lambda}{\mu}\lvert\mathbf{b}\rvert^2+\mathbf{a}\cdot\mathbf{b}\right)}$ $=\dfrac{\lvert\mathbf{a}\rvert^2+\dfrac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert}\mathbf{a}\cdot\mathbf{b}}{\dfrac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert}\lvert\mathbf{b}\rvert^2+\mathbf{a}\cdot\mathbf{b}}$ $=\dfrac{\dfrac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert}\left(\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert+\mathbf{a}\cdot\mathbf{b}\right)}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert+\mathbf{a}\cdot\mathbf{b}}$ $=\dfrac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert}$ deduced From the above result, $\lvert\mathbf{b}\rvert(\mathbf{r}\cdot\mathbf{a})=\lvert\mathbf{a}\rvert(\mathbf{r}\cdot\mathbf{b})$ Let θ_1 and θ_2 be the angle between $\mathbf{a}$ and $\mathbf{r}$, $\mathbf{b}$ and $\mathbf{r}$ respectively. Then $\lvert\mathbf{b}\rvert\left(\lvert\mathbf{r}\rvert\lvert\mathbf{a}\rvert\cos\theta_1\right)=\lvert\mathbf{a}\rvert\left(\lvert\mathbf{r}\rvert\lvert\mathbf{b}\rvert\cos\theta_2\right)$ $\cos\theta_1=\cos\theta_2$ $\theta_1=\theta_2$	[M1] [M1] [M1] [A1]	

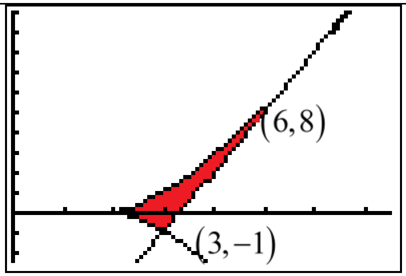
			Since $\cos\theta$ is a 1-1 function from $0 \le \theta \le \pi$. $\therefore$ the line *OR* bisects angle *AOB* (shown).		
	• To determine the algebraic relationship of the 3 planes. **[HOT]**	**(b)(i)**	$x + 2y + 3z = 1$ —(1) $2x + y - 3z = 5$ —(2) $2x - y - 9z = 7$ —(3) Using G.C.: Line of intersection : $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}, \ \lambda \in \Re$	**[M1]** **[A1]**	
		(ii)	If the three planes don't intersect, then the plane π_3 does not intersect the line of intersection. Thus the plane is parallel to the line and thus the normal is perpendicular to the direction vector of the line. $\begin{pmatrix} 2 \\ a \\ b \end{pmatrix} \bullet \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix} = 0 \Rightarrow 6 - 3a + b = 0$ $\Rightarrow 3a - b = 6$ Since (3,–1, 0) cannot lie on π_3. $\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ a \\ b \end{pmatrix} \ne 5 \Rightarrow 6 - a \ne 5 \Rightarrow a \ne 1$ Thus $b = 3a - 6, \quad a \in \mathbb{R} \setminus \{1\}$.	**[M1]** **[A1]** **[M½]** **[A½]**	

10	Integration/ Definite Integrals			
	Assessment Objectives	**Solution**		**Feedback**
	• To use a given substitution to simplify an integral before integration.	**(a)** $x = 2\sec\theta$ $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\sec\theta\tan\theta$ $\int \frac{1}{x^2\sqrt{x^2-4}}\,\mathrm{d}x$ $= \int \frac{1}{4\sec^2\theta\sqrt{4(\sec^2\theta-1)}}\,2\sec\theta\tan\theta\,\mathrm{d}\theta$ $= \int \frac{1}{2\sec\theta\sqrt{4\tan^2\theta}}\tan\theta\,\mathrm{d}\theta$ $= \int \frac{1}{2\sec\theta\left(2\tan\theta\right)}\tan\theta\,\mathrm{d}\theta$ $= \int \frac{1}{4\sec\theta}\,\mathrm{d}\theta$ $= \frac{1}{4}\int \cos\theta\,\mathrm{d}\theta$ $= \frac{1}{4}\sin\theta + C$ $= \frac{\sqrt{x^2-4}}{4x} + C$ **Note:** $x = 2\sec\theta \Rightarrow \cos\theta = \frac{2}{x}$ 	 **[M1]** **[M1]** **[M1]** **[M1]** **[A1]**	

	• To find the volume of revolution about the *x*-axis. • To use the integrals of $\dfrac{1}{a^2+x^2}$	**(b)** $V=\pi\int_{-\sqrt{\frac{3}{2}}}^{-\frac{1}{\sqrt{2}}}\left(\frac{1}{\sqrt{1+2x^2}}\right)^2\,dx$ $=\pi\int_{-\sqrt{\frac{3}{2}}}^{-\frac{1}{\sqrt{2}}}\frac{1}{1+2x^2}\,dx$ $=\frac{\pi}{2}\int_{-\sqrt{\frac{3}{2}}}^{-\frac{1}{\sqrt{2}}}\frac{1}{\frac{1}{2}+x^2}\,dx$ $=\frac{\pi}{2}\left[\frac{1}{\frac{1}{\sqrt{2}}}\tan^{-1}\left(\frac{x}{\frac{1}{\sqrt{2}}}\right)\right]_{-\sqrt{\frac{3}{2}}}^{-\frac{1}{\sqrt{2}}}$ $=\frac{\pi}{2}\left[\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)\right]_{-\sqrt{\frac{3}{2}}}^{-\frac{1}{\sqrt{2}}}$ $=\frac{\pi}{2}\left[\sqrt{2}\tan^{-1}(-1)-\sqrt{2}\tan^{-1}\left(-\sqrt{3}\right)\right]$ $=\frac{\pi}{2}\left[\sqrt{2}\left(-\frac{\pi}{4}\right)-\sqrt{2}\left(-\frac{\pi}{3}\right)\right]$ $=\frac{\sqrt{2}}{2}\pi^2\left[-\frac{1}{4}+\frac{1}{3}\right]$ $=\frac{\sqrt{2}}{24}\pi^2$	**[M1]** **[M1]** **[M1]** **[A1]**	•

11 Applications of Differentiation

Assessment Objectives	Solution		Feedback
• Sketch the graph of parametric equations.	**(i)**  Shape [**B½**] Intercept [**B½**]		
• Find the equation of tangent	**(ii)** $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}=3t^2\times\frac{1}{2t}=\frac{3}{2}t$ When $t=2$, $x=6$ $y=8$ $\frac{\mathrm{d}y}{\mathrm{d}x}=3$ Equation of tangent: $y-8=3(x-6)$ $y=3x-10$	**[M1]** **[A1]** **[A1]**	
• Find intersection between a line and parametric equation	**(iii)** Let $t=q$ at the point of intersection Q. Then the point of intersection is (q^2+2,q^3). Since Q lies on line l, $q^3=3(q^2+2)-10$ $q^3-3q^2+4=0$ Since $q=2$ is a solution to the cubic equation, $q^3-3q^2+4=(q-2)(q^2+aq-2)$ Comparing the coefficient of q^2, $-2q^2+aq^2=-3q^2$ $\therefore\ a=-1$	 **[M1]** **[M1]**	

		$q^3-3q^2+4=0$ $(q-2)(q^2-q-2)=0$ $(q-2)(q-2)(q+1)=0$ $q=2\,(\text{rejected since that is point P})$ or $q=-1$ $\therefore\ Q(3,-1)$	**[M1]** **[A1]**	
	• Find area of parametric curve. **[HOT]**	**(iv)**  Area $=$ Area of trapezium $-\int_{-1}^{8} x\,\mathrm{d}y$ $=\frac{1}{2}(3+6)(8-(-1))-\int_{-1}^{8} x\,\mathrm{d}y$ Consider $\int_{-1}^{8} x\,\mathrm{d}y=\int_{-1}^{2}(t^2+2)(3t^2)\,\mathrm{d}t$ $=\int_{-1}^{2} 3t^4+6t^2\,\mathrm{d}t$ $=\left[\frac{3}{5}t^5+2t^3\right]_{-1}^{2}$ $=\left[\frac{3}{5}(32)+2(2^3)\right]-\left[\frac{3}{5}(-1)+2(-1)\right]$ $=37\frac{4}{5}\text{ units}^2$ Area $=40\frac{1}{2}-37\frac{4}{5}=2\frac{7}{10}\text{ units}^2$	**[M1]** **[M1]** **[M1]** **[A1]**	

<table>
<tr><td>12</td><td colspan="4">Complex Numbers</td></tr>
<tr><td></td><td>Assessment Objectives</td><td colspan="2">Solution</td><td>Feedback</td></tr>
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- To find the modulus and argument of Quotient for a complex number.
- Able to express complex number in x+i y and trigonometric form, hence find tan(arg$z$).

</td><td>

(a)

$$|z|=\left|\frac{(1+i)^3}{\sqrt{3}-i}\right|=\frac{\sqrt{2}^3}{2}$$

$$=\sqrt{2}$$

$$\arg\frac{(1+i)^3}{\sqrt{3}-i}=3\arg(1+i)-\arg(\sqrt{3}-i)$$

$$=3\left(\frac{\pi}{4}\right)-\left(-\frac{\pi}{6}\right)=\frac{11\pi}{12}$$

$$\therefore z=\frac{(1+i)^3}{\sqrt{3}-i}=\sqrt{2}\,e^{i\left(\frac{11\pi}{12}\right)}=\sqrt{2}\left(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12}\right)$$

$$\frac{(1+i)^3}{\sqrt{3}-i}=\frac{2(-1+i)}{\sqrt{3}-i}=\frac{2(-1+i)}{\sqrt{3}-i}\times\frac{\sqrt{3}+i}{\sqrt{3}+i}$$

$$=\frac{-(1+\sqrt{3})}{2}+\frac{(\sqrt{3}-1)i}{2}$$

$$\therefore\quad \frac{-(1+\sqrt{3})}{2}+\frac{(\sqrt{3}-1)i}{2}=\sqrt{2}\left(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12}\right)$$

$$\Rightarrow \sqrt{2}\cos\frac{11\pi}{12}=-\frac{-(1+\sqrt{3})}{2}$$

$$\sqrt{2}\sin\frac{11\pi}{12}=\frac{(\sqrt{3}-1)}{2}$$

$$\therefore \tan\frac{11\pi}{12}=-\frac{\sqrt{3}-1}{\sqrt{3}+1}=\sqrt{3}-2$$

</td><td>

[M1]

[A1]

[M1]

[A1]

[M1]

[A1]

[M½]

[M½]

[A1]

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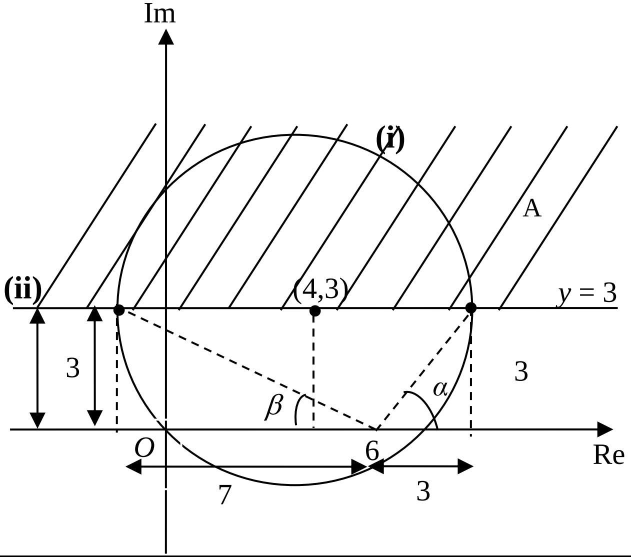
	• To draw the locus of circle and line of perpendicular bisector • Shade the required region and find the points of intersection using various method. • To express the range of the required arg z using the correct diagram. **[HOT]**	**(b)(i)**  		Circle **[B1]** Centre of circle & passes thru origin **[B1]** Perpendicular bisector **[B1]** Shading **[B1]**
		(ii) Min arg$(z-6)$ = $\tan^{-1}\left(\frac{3}{3}\right) = \tan^{-1} 1$ $= \frac{\pi}{4} = 0.785$ Max arg$(z-6)$ = $\pi - \beta = \pi - \tan^{-1}\left(\frac{3}{7}\right)$ $= 2.737$ **<u>Alternatively, we can locate the points of intersections then proceed to obtain the required argument:</u>** Equation of circle: $(x-4)^2 + (y-3)^2 = 25$ —(1) Equation of perpendicular bisector: $y = 3$ —(2) Substitute (2) into (1): $(x-4)^2 = 25$ $x - 4 = \pm 5$ $x = 9$ or -1 Points of intersection are (9, 3) and (–1, 3).	**[M1]** **[A1]** **[M1]** **[A1]**	