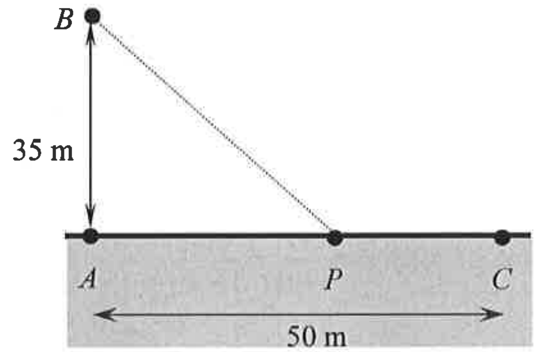
1 Without the use of a calculator, solve the inequality $\frac{5x}{x-2} \geq 3$. Hence, find the range of values of x that satisfy $\frac{5|x|}{|x|-2} \geq 3$. [5]

2

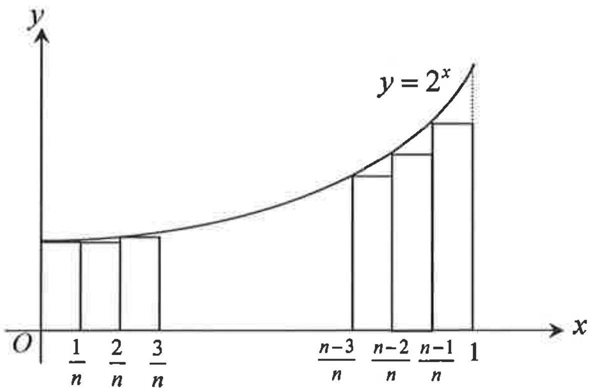


In the above diagram, not drawn to scale, a man in a boat at B is 35 m from A, the nearest point on a straight shore AC. He intends to disembark from his boat at the point P and runs along the shore to point C which is 50 m from A. He can row at $3\ \mathrm{m\,s^{-1}}$ and run at $4\ \mathrm{m\,s^{-1}}$ along straight paths.

(i) If x denote the distance, in metres, between A and P, and t denote the total time, in seconds, required to travel from B to C, show that $t = \frac{\sqrt{1225+x^2}}{3} + \frac{50-x}{4}$. [2]

(ii) Find the exact value of x such that the man is able to travel from B to C in the shortest possible time. You may assume that the time taken to disembark from the boat is negligible. [3]

3 The graph of $y = 2^x$ for $0 \leq x \leq 1$, is shown in the diagram below. Rectangles, each of width $\frac{1}{n}$, where n is an integer are drawn under the curve.



(i) Show that the total area of all the n rectangles is $\frac{1}{n\left(2^{\frac{1}{n}}-1\right)}$. [2]

(ii) By considering the area of the region bounded by the curve $y = 2^x$, $x = 1$ and the axes, briefly explain why $\ln 2 < n\left(2^{\frac{1}{n}}-1\right)$. [3]

[Turn over

4 Two functions f and g are defined by

$$\mathrm{f}: x \mapsto x^2 + 2x, \qquad x \le 0;$$
$$\mathrm{g}: x \mapsto -\ln(2+x), \qquad x \ge 0.$$

(i) Give a reason why the inverse of f does not exist. [1]

(ii) Only one of the functions fg and gf exists. Determine which one of them exists, and state its rule, domain and range. Explain why the other does not exist. [5]

5 The points A and B relative to the origin O have position vectors $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $-4\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ respectively. The point P lies on line AB such that $\dfrac{AP}{PB} = \dfrac{\lambda}{1-\lambda}$.

(i) Show that $\overrightarrow{OP} = (1-5\lambda)\mathbf{i} + (2+3\lambda)\mathbf{j} + (4\lambda-2)\mathbf{k}$. [1]

(ii) Given further that C is a point with position vector $-5\mathbf{i}+16\mathbf{j}-2\mathbf{k}$ and that $\overrightarrow{OP} = \mu\overrightarrow{OC}$, show that $\mu = \dfrac{1}{5}$ and find the value of λ. [3]

(iii) Hence, or otherwise, determine the area of triangle OAP, leaving your answer correct to 3 decimal places. [2]

6 (i) The set of points P in an Argand diagram represents the complex number z that satisfies $|z-2-\mathrm{i}| = 2$. Sketch the locus of P. [1]

(ii) The set of points Q represents another complex number w given by $\arg(w-2+3\mathrm{i}) = \theta$ where $-\pi < \theta \le \pi$. Give a geometrical description of the locus of Q. [1]

(a) Find the range of values of θ such that the locus of Q meets the locus of P more than once. [2]

(b) In the case where $\theta = \tan^{-1}\left(\dfrac{3}{4}\right)$, find the least value of $|z-w|$ in exact form. [3]

7 The number of Bye-Bye Panda due to birth and death in a protected bamboo environment is modelled by the differential equation $\dfrac{\mathrm{d}S}{\mathrm{d}t} = kS(k-S)$ where S is the number of pandas at time t years and k is a constant. Given that $k > S$, show that the differential equation has general solution $S = \dfrac{kA}{A+\mathrm{e}^{-k^2t}}$, where A is an arbitrary constant. [5]

Hence, find the limiting value of the number of Bye-Bye Panda in the bamboo environment and explain the significance of k. [2]

[Turn over

8 The sequence u_n is defined by the recurrence relation

$$u_1 = 1 \text{ and } u_{n+1} = \frac{u_n}{u_n + 2} \text{ for } n \geq 1.$$

(i) Write down the values of u_2, u_3 and u_4. Hence, make a conjecture for u_n in terms of n. [2]

(ii) Use mathematical induction to prove that your conjecture is true for all positive integers n. [5]

9 Damian deposits P dollars in POSD bank at the beginning of a year. The deposit earns an interest rate of $x\%$ per annum compounded annually. At the end of each year, $\frac{1}{3}$ of the amount in the account (including principal and interest) is withdrawn and the remainder is re-deposited at the same rate. Let u_n denote the amount of money withdrawn at the end of the n^{th} year.

(i) Express u_1 and u_2 in terms P and x and hence show that $u_3 = \frac{4}{27}P\left(1 + \frac{x}{100}\right)^3$. [3]

(ii) Given that u_1, u_2, u_3, ... form a geometric progression, find the common ratio in terms of x. [1]

(iii) Suppose $u_3 = \frac{27}{128}P$, find the value of x. Given further that $P = 5000$, find the total amount of money withdrawn at the end of the first 5 years. Give your answer correct to the nearest integer. [4]

10 A line l is given by the equation $\mathbf{r} = 2\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$, $\lambda \in \mathbb{R}$ and the point P has position vector $2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ relative to the origin. N is a point on l such that the line PN is perpendicular to l.

(i) Show that $\overrightarrow{PN} = \frac{1}{7}(5\mathbf{i} + 17\mathbf{j} + 13\mathbf{k})$. [2]

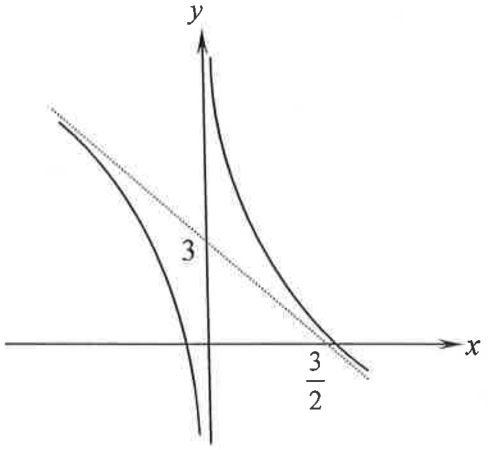
(ii) The plane Π_1 has equation $\mathbf{r}\cdot(3\mathbf{i} + \mathbf{k}) = 7$. Verify that l lies in Π_1 and find the perpendicular distance from point P to Π_1. [4]

(iii) A second plane Π_2 contains l and P. Using your answers above, or otherwise, determine the acute angle between Π_1 and Π_2. [2]

(iv) A third plane Π_3 has equation $\mathbf{r}\cdot(-\mathbf{j} + 5\mathbf{k}) = 22$. Determine the position vector of the point of intersection between planes Π_1, Π_2 and Π_3. [2]

[Turn over

11



The diagram above shows the graph of the function $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = \dfrac{ax^2 + bx + c + 1}{x + d}$, $a, b, c, d \in \mathbb{R}$. The oblique asymptote intersects the axes at $x = \dfrac{3}{2}$ and $y = 3$. Find the values of a, b and d. By considering $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, show that $c > -1$. [5]

Sketch, on separate diagrams, the graphs of

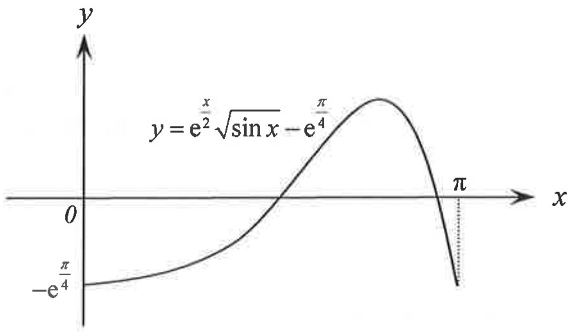
(i) $y = \mathrm{f}'(x)$; [3]

(ii) $y = \dfrac{4ax^2 - 2bx + c + 1}{d - 2x}$, [3]

indicating clearly any asymptotes.

12 **(i)** Use integration by parts to show that $\int \mathrm{e}^x \sin x \, \mathrm{d}x = \dfrac{\mathrm{e}^x}{2}(\sin x - \cos x) + c$, where c is an arbitrary constant. [4]

(ii)



The diagram above shows the graphs of a curve C given by $y = \mathrm{e}^{\frac{x}{2}}\sqrt{\sin x} - \mathrm{e}^{\frac{\pi}{4}}$ for $0 \le x \le \pi$.

(a) The region R is bounded by the x-axis and the part of C between $x = 0$ and $x = \dfrac{5}{2}$. Find, correct to 3 significant figures, the area of region R. [4]

(b) The region S is bounded by C and the line $y = -\mathrm{e}^{\frac{\pi}{4}}$. Write down the equation of the curve obtained when C is translated $\mathrm{e}^{\frac{\pi}{4}}$ units in the positive y-direction. Hence, show that the volume V of the solid formed when S is rotated 2π radians about the line $y = -\mathrm{e}^{\frac{\pi}{4}}$ is given by $V = a\left(\mathrm{e}^{\pi} + b\right)$, where a and b are exact constants to be determined. [3]

[Turn over

13 **(a)** The complex numbers z_1 and z_2 satisfy the equation $\frac{4}{z} - z = 2\mathrm{i}$. Given that $\mathrm{Re}(z_1) > \mathrm{Re}(z_2)$, find z_1 in the form $x + \mathrm{i}y$, where x and y are exact real constants to be determined. [2]

By expressing z_1 in polar form, or otherwise, find the least possible positive integer n such that $(z_1)^n$ is real. [3]

(b) **(i)** Show that $1 + \mathrm{e}^{\mathrm{i}\alpha} = 2\cos\left(\frac{\alpha}{2}\right)\mathrm{e}^{\mathrm{i}\left(\frac{\alpha}{2}\right)}$ for all $\alpha \in \mathbb{R}$. [2]

(ii) Find the fourth roots of the complex number $8\sqrt{2} - \left(8\sqrt{2}\right)\mathrm{i}$. [2]

(iii) By solving the equation $(w-2)^4 = 8\sqrt{2} - \left(8\sqrt{2}\right)\mathrm{i}$, prove that $4\cos\left(-\frac{9\pi}{32}\right)\mathrm{e}^{\mathrm{i}\left(\frac{-9\pi}{32}\right)}$ is a root to the equation and find the other roots, leaving your answers in similar form. [3]

***** **END OF PAPER 1** *****